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Andrea Bartezzaghi, Luca Dedè, Alfio Quarteroni

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Isogeometric Analysis of Geometric Partial Differential Equations

Andrea Bartezzaghi[a,*], Luca Dedè[a], Alfio Quarteroni[a,b]

[a] *CMCS – Chair of Modeling and Scientific Computing, MATHICSE – Mathematics Institute of Computational Science and Engineering, EPFL – École Polytechnique Fédérale de Lausanne, Station 8, Lausanne, CH–1015, Switzerland*

[b] *MOX – Modeling and Scientific Computing, Department of Mathematics, Politecnico di Milano, Piazza L. da Vinci 32, Milano, 20133, Italy (on leave)*

Abstract

We consider the numerical approximation of geometric Partial Differential Equations (PDEs) defined on surfaces in the 3D space. In particular, we focus on the geometric PDEs deriving from the minimization of an energy functional by L^2-gradient flow. We analyze two energy functionals: the area, which leads to the mean curvature flow, a nonlinear second order PDE, and the Willmore energy, leading to the Willmore flow, a nonlinear fourth order PDE. We consider surfaces represented by single-patch tensor product NURBS and discretize the PDEs by means of NURBS-based Isogeometric Analysis in the framework of the Galerkin method. To approximate the high order geometric PDEs we use high order continuous NURBS basis functions. For the time discretization of the nonlinear geometric PDEs, we use Backward Differentiation Formulas (BDF) with extrapolation of the geometric quantities involved in the weak formulation of the problem; in this manner, we solve a linear problem at each time step. We report numerical results concerning the mean curvature and Willmore flows on different geometries of interest and we show the accuracy and efficiency of the proposed approximation scheme.

Keywords: Geometric Partial Differential Equation, Surface, High Order, Isogeometric Analysis, Mean Curvature Flow, Willmore Flow

1. Introduction

Geometric Partial Differential Equations (PDEs) describe the evolution of the geometrical domain in which these equations are set [1]; such problems are usually defined on surfaces in 3D and the surface itself represents the unknown of the geometric PDE. The computational domain evolves in time, or pseudo-time, according to geometric quantities of interest, such as the curvature of the

*Corresponding author.
Email address: andrea.bartezzaghi@epfl.ch (Andrea Bartezzaghi)