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Least-squares Mixed Generalized Multiscale Finite Element Method *

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ABSTRACT

In this paper, we present an approximation of elliptic problems with multiscale and high-contrast diffusion coefficients. A mixed formulation is considered such that both pressure and velocity are approximated simultaneously. This formulation arises naturally in many applications such as flows in porous media. Due to the multiscale nature of the solutions, using model reduction is required to efficiently obtain approximate solutions. There are many multiscale approaches for elliptic problems in mixed formulation. These approaches include numerical homogenization and mixed multiscale finite element method, which aim to obtain a coarse accurate representation of the velocity without using an accurate representation for pressure. It has been a challenging task to construct a method giving accurate representation for both pressure and velocity. The goal in this paper is to construct multiscale basis functions for both pressure and velocity. We will apply the framework of Generalized Multiscale Finite Element Method (GMsFEM), and design systematic strategies for the construction of basis. The construction involves snapshot spaces and dimension reduction via local spectral problems. The mixed formulation is minimized in the sense of least-squares. The compatibility condition for multiscale finite element spaces of the pressure and velocity is not required in the least-squares mixed form. This gives more flexibility for the construction of multiscale basis functions for velocity and pressure. Convergence analysis is carried out for the least-squares mixed GMsFEM. Several numerical examples for various permeability fields are presented to show the performance of the presented method. The numerical results show that the least-squares mixed GMsFEM can give accurate approximation for both pressure and velocity using only a few basis functions per coarse element.

keywords least-squares mixed formulation, generalized multiscale finite element method, flux correction, high-contrast coefficients

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