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Modeling and simulations for fluid and rotating structure interactions

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Abstract

In this paper, we study a dynamic fluid–structure interaction (FSI) model for an elastic structure immersed and spinning in the fluid. To describe the motion of a rotating elastic structure, we develop a linear constitutive model, that is suitable for the application of the arbitrary Lagrangian–Eulerian (ALE) method in FSI simulations. Additionally, a new ALE mapping method is designed to generate the moving fluid mesh while the deformable structure spins in a non-axisymmetric fluid channel. The structure velocity is adopted as the principle unknown to form a monolithic saddle-point system together with fluid velocity and pressure. Using the mixed finite element method and Newton's linearization, we discretize the nonlinear saddle-point system, and prove that the discrete saddle-point system is well-posed. The developed methodology is applied to a self-defined elastic structure and a realistic hydro-turbine under a prescribed angular velocity. Numerical validation is also conducted to demonstrate the accuracy of the models and the numerical methods.

Keywords: Fluid-rotating structure interaction; Arbitrary Lagrangian Eulerian (ALE) method; Monolithic algorithm; Mixed finite element method; Linear elasticity; Master–slave relations

1. Introduction

The fluid–structure interaction (FSI) problem remains one of the most challenging in computational mechanics and computational fluid dynamics. Researchers have conducted various studies on certain types of FSI problems (such as the fluid-rigid body interaction [1–5], fluids with a non-rotational structure [6–11], and FSI with a fixed fluid domain [12–15]). However, there is a dearth of practical models on FSI problems involving a structure that rotates and deforms, i.e., an elastic rotor. The development of mathematical models and numerical methodologies is critical in practice for large-scale advanced FSI simulations involving an elastic rotor, in order to guide the design, evaluation,

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and prediction of various applications, such as hydro-turbines, jet engines, and artificial heart pumps. Therefore, it is of great significance to develop an efficient and accurate mathematical model and numerical method to handle fluid–structure interactions involving a rotational and deformable structural motion.

However, numerical simulations of fluids interacting with a rotating structure can be challenging. The difficulties associated with simulating the rotational structure in FSI stem from the fact that the simulations rely on the coupling of two distinct descriptions: the Lagrangian description for the solid and the Eulerian coordinate for the fluid. The arbitrary Lagrangian Eulerian (ALE) method [6,16–19] copes with this difficulty by adapting the fluid mesh to accommodate the deformations of the solid on the interface. When the ALE method is used, the meshes of the fluid and structure conform on the interface if the Lagrangian structure mesh is moved to the Eulerian one following ALE mapping. This is important as the degrees of freedom on the interface are naturally shared by both the fluid and the structure, which facilitates the implementation of the discretization proposed herein. However, the ALE method has a severe drawback; i.e., when the structure has a large displacement or deformation, it is very likely that ALE mapping will distort the fluid mesh. Even the most advanced and most finely tuned ALE-based scheme cannot perform well without re-meshing. And, if re-meshing is used to produce the fluid mesh, then the number of mesh nodes and/or elements over different time levels can no longer be guaranteed to be the same. Thus, interpolations of variables between every two adjacent time steps are unavoidable, resulting in a time-consuming and even unstable geometrical process, especially in high-dimension cases. In fact, it is difficult to directly apply the ALE method to fluid-rotating structure interaction problems.

Many approaches have been proposed to deal with rotational structures in FSI problems. Several of these approaches model the wind turbine rotor [20–24] by coupling the finite element method (FEM) for fluid dynamics, isogeometric analysis (IGA) for structural mechanics, and the non-conforming discretization on the interface between fluid and structure. In order to apply ALE, these approaches introduce an artificial cylindrical buffer zone to enclose the rotor; let the mesh of the sliding cylindrical interface weakly enforce the continuity of the solution fields; and introduce extra unknowns and/or penalties to reinforce the continuity on the interface via, for example, the Lagrange multiplier or the discontinuous Galerkin (DG) method. The shear-slip method [25,26,20] was introduced to locally reconnect the mesh in order to maintain its the quality when the structure is undergoing translation or rotation. There are also some interesting ALE remeshing techniques that are able to deal with large structure displacement. For example, the fixed-mesh ALE method [27] and the universal mesh method [28] locally rebuild the mesh in order to accommodate the arbitrary motion of the interface.

In this paper, we develop a new ALE method in order to produce a body-fitted moving fluid mesh that conforms with the rotational and deformable structure mesh on the interface. And, to make our ALE method work for an elastic rotor immersed in a fluid, we first derive a linear structure equation that involves the rotational matrix from the nonlinear structure model based on decomposing the structure displacement into two components: rotation and deformation. In addition, we define an artificial cylindrical buffer zone in the fluid domain to enclose the elastic rotor and rotate together on the same axis of rotation with the same angular velocity. If the fluid channel is non-axisymmetric, we need to determine the relative motion information between the rotational fluid subdomain (the cylindrical buffer zone) and the fixed fluid subdomain (the rest of the fluid domain) by matching the grid on the sliding interface and defining them in our new ALE mapping. Finally, we develop a very stable and easily attainable ALE method designed to generate a rotational and deformable fluid mesh that matches the structure mesh on the interface.

Next, with the velocity instead of the displacement as the principle unknown of the structure, we define a monolithic saddle-point system for the studied FSI problem, and further a monolithic algorithm to solve the coupled fluid and structure equations. Moreover, we are able to prove the well-posedness of the discrete monolithic linear saddle-point system resulting from mixed finite element discretization and Newton's method. Numerical experiments are carried out for a self-defined elastic rotor and a realistic hydro-turbine to illustrate that our developed structure model and ALE-based monolithic method are efficient and stable for the FSI problem involving an elastic rotor. A numerical validation is also conducted to demonstrate the consistency of our rotational structure model with respect to the different structure parameters.

The paper is organized as follows. In Section 2, we define the general governing equations, interface conditions, and boundary conditions of the FSI problem, and their weak formulations along with the ALE techniques, and then we introduce the monolithic weak formulation of FSI. In Section 3, we develop an approximate formulation of the constitutive equation for the rotating structure. Then, in Section 4, we define a new ALE mapping for an elastic rotor immersed in the fluid, and we define a monolithic numerical discretization in Section 5, where we also analyze the