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ScienceDirect
Comput. Methods Appl. Mech. Engrg. 311 (2016) 815–837
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

# A local crack-tracking strategy to model three-dimensional crack propagation with embedded methods

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Received 18 July 2016; received in revised form 13 September 2016; accepted 19 September 2016
Available online 29 September 2016

## Highlights

- Developed a crack-tracking approach to propagate embedded failure surfaces in 3D.
- The effect of Dirichlet boundary conditions on the crack-tracking method is closely examined.
- Several numerical examples are conducted to compare the method with more traditional alternatives.

## Abstract

We develop a local, implicit crack tracking approach to propagate embedded failure surfaces in three-dimensions. We build on the global crack-tracking strategy of Oliver et al. (Int J. Numer. Anal. Meth. Geomech., 2004; 28:609–632) that tracks all potential failure surfaces in a problem at once by solving a Laplace equation with anisotropic conductivity. We discuss important modifications to this algorithm with a particular emphasis on the effect of the Dirichlet boundary conditions for the Laplace equation on the resultant crack path. Algorithmic and implementational details of the proposed method are provided. Finally, several three-dimensional benchmark problems are studied and results are compared with available literature. The results indicate that the proposed method addresses pathological cases, exhibits better behavior in the presence of closely interacting fractures, and provides a viable strategy to robustly evolve embedded failure surfaces in 3D.


*Keywords:* 3D fracture; Embedded cracks; Crack-tracking; X-FEM; G-FEM

## 1. Introduction

In recent years, the finite element community has focused attention on a class of embedded interface methods that allow cracks to be oriented arbitrarily with respect to the underlying finite element mesh. Both the Generalized/eXtended Finite Element Methods (G/X-FEM) [1–4] and the Strong Discontinuity Approaches (also known as embedded FEM or E-FEM) [5–8], facilitate the treatment of cracks as arbitrary interfaces by enhancing the

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http://dx.doi.org/10.1016/j.cma.2016.09.018

kinematics of the underlying mesh. By treating cracks as arbitrary interfaces, these methods offer the potential for addressing the pathological mesh-dependence of the interface element approaches [9,10]. However, since the cracks are now arbitrary with respect to the underlying volume mesh, a tracking mechanism separate from the bulk mesh needs to be introduced to locate the crack surface within the finite element mesh.

Within the embedded finite element methods (G/X-FEM and E-FEM), crack-tracking algorithms can be broadly classified into two major categories: (a) explicit approaches, and (b) implicit approaches. In the explicit methods, cracks are represented as a collection of piecewise segments in 2D, and piecewise triangular and quadrangular surfaces in 3D. Early efforts in this direction discretized the evolving crack surface through a $C^0$ continuous surface formed from a union of the triangles and quadrilaterals that separate a cracked tetrahedral element into two (see Areias and Belytschko [11], Gasser and Holzapfel [12]). However, with these approaches in 3D, one needs to modify the normal to the local crack plane in an *ad hoc* manner if a globally continuous crack path is desired. A more general methodology is to define two completely independent meshes, namely the underlying volume mesh and an independent triangulation of the crack surface (see for *e.g.* [13–15]). The reader is referred to Garzon et al. [16] and the references therein for a detailed description of these approaches and the current state-of-the-art. While these approaches have the advantage that the element size has no bearing on the accuracy of the crack surface representation, they also require a very specific machinery to handle the computational geometry challenges not available in most general purpose finite element software.

By contrast, in implicit approaches, the crack surface is represented by a zero iso-surface of a signed-distance field associated with the nodes of the underlying finite element mesh. Each crack is represented by two orthogonal level set functions, one associated with the crack surface and the second associated with the crack front such that the intersection of the zero iso-surfaces of these two functions precisely locates a crack front (see Möes et al. [17]). In addition, Hamilton–Jacobi type equations are solved over whole or part of the domain using finite difference methods (see Gravouil et al. [18]) or Fast Marching Methods (see Sukumar et al. [19,20]) to evolve these level set functions as the crack propagates. More recently, improvements to these algorithms have also been proposed in Duflot [21] and Colombo and Massin [22]. Fries et al. [23] have also developed hybrid implicit–explicit approaches that combine the advantages of the aforementioned methods.

In the context of embedded finite element methods, Oliver et al. [24] developed an alternate implicit strategy that solves a Laplace equation with an anisotropic conductivity tensor as a global crack-tracking methodology. In this methodology, all possible crack paths are tracked at once through a nodally defined propagation field. Any given crack then corresponds to an iso-surface value of this field and is easily identified. The idea is remarkably simple yet retains all the advantages associated with the level-set methods and is arguably more suited to being integrated in an existing general purpose finite element framework. Since the original paper by Oliver et al. [24], the approach has been extensively used in both the E-FEM and X-FEM frameworks in several studies (see [25–32]). In 2D, a comparison between the global and explicit crack tracking approaches is presented in Dumstorff and Meschke [33,34].

Jäger et al. [30–32] in a series of articles present a thorough analysis of the global crack tracking approach when used in conjunction with the phantom node method to model crack propagation in 3D. They compare and contrast the method's performance with the alternatives available and highlight the method's promise to model arbitrary crack propagation problems in 3D. However, they also report that the results of the global approach are sensitive to the Dirichlet boundary conditions applied to the crack-tracking problem. In Jäger et al. [32], they propose geometry-based, and mesh-based strategies to enforce Dirichlet boundary conditions to circumvent this sensitivity.

In the current work, we revisit this sensitivity analysis and demonstrate the spurious behavior that could result for certain choices of Dirichlet boundary conditions for the global crack-tracking method. To resolve this difficulty, we propose an alternative approach that solves the anisotropic Laplace equation in a more localized domain just ahead of the crack front. It is noteworthy that similar ideas have been proposed earlier in Feist and Hofstetter [26] and Armero and Kim [27]. In the *Partial Domain Tracking Algorithm* of Feist and Hofstetter [26], the crack-tracking problem is solved for a subset of the domain that is potentially intersected by a crack. Armero and Kim [27] propose an element local solution where the Laplace problem is solved for one front element at a time. However, the approach presented here is distinct for two reasons (a) we demonstrate that a local solution is required *not just* from an efficiency perspective but also to prevent spurious crack path, and (b) the local solution procedure we are advocating involves all elements just ahead of the crack front unlike either of the methods described above. Finally, while we use the Hansbo method [3] to enhance the kinematics of the elements intersected by the discontinuity, the crack-tracking approach presented here is equally applicable to both the X-FEM and E-FEM techniques.