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A Prange-Hellinger-Reissner type finite element formulation for small strain elasto-plasticity

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Highlights

- Mixed Prange-Hellinger-Reissner type variational functional
- Algorithmic consistent treatment of plane stress and plane strain
- Point-wise enforcement of constraints due to elasto-plasticity in contrast to the element-wise treatment proposed by Simo et al. [1989]

Abstract

In this contribution we propose a mixed variational formulation of the Prange-Hellinger-Reissner type for elasto-plasticity at small strains. Here, the displacements and the stresses are interpolated independently, which are balanced within the variational functional by the relation of the elastic strains and the partial derivative of the complementary stored energy with respect to the stresses. For the elasto-plastic material behavior a von Mises yield criterion is considered, where we restrict ourselves w.l.o.g. to linear isotropic hardening. In the proposed formulation we enforce the constraints arising from plasticity point-wise in contrast to the element-wise realization of the plastic return mapping algorithm suggested in Simo et al. [1989]. The performance of the new formulation is demonstrated by the analysis of several benchmark problems. Here, we compare the point-wise treatment of elasto-plasticity with the original element-wise formulation of Simo et al. [1989]. Furthermore, we derive an algorithmic consistent treatment for plane stress as well as for plane strain condition.

Keywords: mixed FEM, elasto-plasticity, Prange-Hellinger-Reissner type functional, algorithmic consistent treatment, plane stress and plane strain

1. Introduction

An important field in finite element design is the improvement of the element performance, which is influenced by the reliability, stability and solution quality. In the past decades, an enormous effort was addressed to the development on finite element methods based on the variational approach going back to Galerkin [1915], which are in general considering the approximation of one field, e.g. the displacements. Further developments also consider additional fields in the variational setup for example in Reissner [1950] (compare also the early works of Hellinger [1914] and Prange [1916]), an independent stress approximation is