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Approximation of constrained problems using the PGD method with application to pure Neumann problems

Kenan Kergrene[1], Serge Prudhomme[1], Ludovic Chamoin[2], and Marc Laforest[1]

[1]*Département de mathématiques et de génie industriel, École Polytechnique de Montréal, Montréal, Québec, Canada, H3T 1J4*
[2]*LMT, ENS Cachan, CNRS, Université Paris-Saclay, 61 Avenue du Président Wilson, 94230 Cachan, France*

Abstract

In this paper we introduce, analyze, and compare several approaches designed to incorporate a linear (or affine) constraint within the Proper Generalized Decomposition framework. We apply the considered methods and numerical strategies to two classes of problems: the pure Neumann case where the role of the constraint is to recover unicity of the solution; and the Robin case, where the constraint forces the solution to move away from the already existing unique global minimizer of the energy functional.

Keywords: Model Reduction, Separation of Variables, Low-rank Approximation, Tensor Product Approximation, Proper Generalized Decomposition (PGD), Constrained Problem, Mixed Formulation

1 Introduction

The need for fast evaluation of surface responses in parametric analyses has spurred the development of novel model reduction methods to construct, in an effective manner, solutions to boundary-value problems. One such method is the Proper Generalized Decomposition (PGD) framework [10, 11], in which the solution is sought numerically using the concept of separation of variables. The PGD approximation scheme allows one to simplify a complex problem into a set of coupled problems, defined with respect to each spatial and/or parametric variable, which can be further decoupled using the so-called Alternated Directions scheme [2, 11]. There exist to date a variety of PGD methods [22], which have been adapted to the nature of the problem at hand and which have been successfully tested on a wide range of applications and model problems, see e.g. [3, 5, 6, 8, 9, 12, 23, 26, 28]. Yet, and to the best of the authors' knowledge, none of these applications include problems subjected to constraints defined on the solution space, except, maybe, the case of the incompressible Navier-Stokes equations, for which the divergence-free constraint is treated using a fractional-step or projection method [14, 15]. We also mention the works presented in [1, 17] and