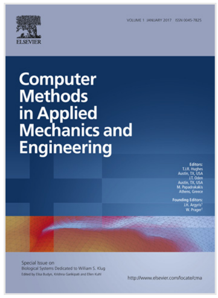

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Subhajit Banerjee, N. Sukumar

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Exact integration scheme for planewave-enriched partition of unity finite element method to solve the Helmholtz problem

Subhajit Banerjee, N. Sukumar*

Department of Civil and Environmental Engineering, University of California, Davis, CA 95616, USA

Abstract

In this paper, we present an exact integration scheme to compute highly oscillatory integrals that appear in the solution of the two-dimensional Helmholtz problem using the planewave-enriched partition of unity finite element method. In the proposed scheme, such oscillatory integrals are computed by a recursive application of the divergence theorem, eventually expressing the integrals in terms of evaluations of the corresponding integrands at the nodes of the finite element mesh. The number of such function evaluations is independent of the wave number k, which permits the scheme to be used for arbitrary high values of k. We consider finite element meshes with unstructured triangular and structured rectangular elements, and present numerical results for three canonical benchmark Helmholtz problems to demonstrate the accuracy and efficacy of the method.

Keywords: Helmholtz equation, planewave enrichment, partition of unity finite element method, highly oscillatory integrals, divergence theorem, exact integration

1. Introduction

In the field of computational acoustics and electrodynamics, the problem of time harmonic wave scattered by bounded obstacles is of central importance. In mathematical physics this phenomenon is condensed in the framework of scattering theory. Scattering theory also finds its use in many applied disciplines such as: medical and seismic imaging, nondestructive testing, radar cross-section prediction, acoustic noise barrier, waveguides, etc. Use of high frequency time-harmonic waves offers distinct advantages in these applications, resulting in a proportionate time-harmonic response. The spatially varying component of this time-harmonic response is governed by the Helmholtz equation.

Consider a problem domain $\Omega \subset \mathbb{R}^2$ with $\boldsymbol{x} \equiv (x, y) \in \Omega$ and time $t \in [0, T] \subset \mathbb{R}^+$. Also let $\psi(\boldsymbol{x}, t) = u(\boldsymbol{x})e^{-i\omega t}$ be a time-harmonic propagating disturbance with $i = \sqrt{-1}$, angular frequency ω, and complex-valued spatially varying component $u(\boldsymbol{x})$. The scalar Helmholtz equation (in homogeneous form) governing $u(\boldsymbol{x})$ is:

$$-\nabla^2 u(\boldsymbol{x}) - k^2 u(\boldsymbol{x}) = 0, \tag{1}$$

where $k = \omega/c$ is the constant wave number with c denoting the wave velocity.

*Corresponding author

Email addresses: `jitbanerjee@ucdavis.edu` (Subhajit Banerjee), `nsukumar@ucdavis.edu` (N. Sukumar)

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