Available online at www.sciencedirect.com
ScienceDirect

Comput. Methods Appl. Mech. Engrg. 317 (2017) 890–926

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Three-dimensional finite elements with embedded strong discontinuities for the analysis of solids at failure in the finite deformation range☆

J. Kim, F. Armero*

Department of Civil and Environmental Engineering, University of California, Berkeley, CA 94720-1710, USA

Received 30 January 2014; received in revised form 6 November 2016; accepted 29 December 2016
Available online 5 January 2017

Abstract

This paper presents the development of new three-dimensional finite elements in the finite deformation range for the numerical modeling of material failure characterized by propagating crack-type surfaces in solids, commonly referred to as strong discontinuities. The singular fields associated with such discontinuous solutions are captured by an element-based enhancement, locally within a multi-scale framework. The complex kinematics arising from the general propagating discontinuity surfaces, especially in the considered finite deformation setting, is captured by the direct identification of the element strain fields associated to the separation modes of the discontinuity, rather than attempting to interpolate directly the corresponding discontinuous deformation fields. To avoid the over-stiff response known as stress-locking, by which the discontinuities cannot open as needed by the material response, the new brick elements consider full linear interpolations of the displacement jumps. This requires a total of nine local parameters for each localized finite element, parameters that are condensed out at the element level. The resulting formulation does not change then neither the number nor the connectivity of the global degrees of freedom employed in the finite element resolution of the underlying global mechanical problem, thus leading to a computationally efficient numerical technique that captures sharply these discontinuous solutions. In particular, the localized dissipative mechanism associated to the material failure is captured objectively through the consideration of the proper cohesive law along the discontinuity surface. Special care is taken to assure the material frame indifference of the final formulation. Complete details of the implementation of the new elements, including the consistent linearization of the governing equations, are also presented. The results obtained in numerical simulations of representative benchmark problems illustrate the performance of the new finite elements with embedded strong discontinuities.

Keywords: Material failure; Finite deformation; Finite element method; Embedded strong discontinuity; Stress locking

---

☆ Research supported by the AFOSR under grant no. FA9550-08-1-0410 with UC Berkeley.

* Corresponding author.
E-mail addresses: jhkim07@berkeley.edu (J. Kim), armero@berkeley.edu (F. Armero).

http://dx.doi.org/10.1016/j.cma.2016.12.038

1. Introduction

The finite element modeling of material failure has received a great deal of attention due to its practical applications in the design and understanding of structural and mechanical systems at the macroscopic level, the scale of interest in this work. Of particular interest are the so-called localized failures, defined broadly by those failures that can be efficiently modeled by a propagating surface exhibiting discontinuous displacements in the deformation of the solid or structure. Clear examples are cracks in brittle material, but also shear bands in ductile failures can be treated in this way due to their different scale in terms of their thickness.

Different approaches can be found in the literature to address this challenging problem. Lately the focus has been in the actual representation of the failure surfaces and their associated localized dissipative mechanism (basically a cohesive law relating the traction to the displacement jump), rather than different considerations at the constitutive level of a continuum model. For example, the modeling of cracks through the so-called cohesive elements [1–3], with the failure surfaces captured along element boundaries, is a clear example of this approach. However, the restriction of the crack paths by the finite element mesh in this framework leads to final solutions dependent on the mesh alignment, requiring adaptive refinement techniques; see e.g., [4–7].

The theoretical characterization of such discontinuous solutions, the so-called strong discontinuities (i.e., jumps in the displacement field with singular strain fields), was considered in [8,9], and references therein, for analyses in the infinitesimal and finite deformation ranges, respectively. These theoretical characterizations and, again, the need to arrive at numerical techniques able to handle accurately and efficiently the large-scale mechanical and structural systems of interest have led to the development of numerical methods for the capture of strong discontinuities, focusing on finite elements that accommodate the discontinuous solutions in the element interiors. In this context, the so-called finite elements with embedded strong discontinuities (sometimes referred to in short by E-FEM), the focus of this work, tackle this challenging issue locally at the element level, in the element's interior, and independent from element to element as the discontinuity surface propagates through the finite element mesh. This locality results in the possibility of eliminating the local degrees of freedom defining the enhancements (the enhanced parameters, usually corresponding to the displacement jumps across the strong discontinuity) at the element level, thus not requiring the consideration of extended algebraic systems of equations in the solution of the large-scale mechanical problem, in terms of the number and connectivity of the global degrees of freedom (usually the nodal displacements) employed in the resolution of this problem. See [10,8,9,11–14] for different variations of these ideas.

In fact, it is shown in [15,16] that the local character of this approach can be viewed as a direct consequence of a multi-scale treatment of the different length scales noted above involved in the phenomenon of material failure, allowing the overall problem to be represented separately in the large and small scales. This strategy results at the continuum level in the standard smooth mechanical problem in the large scale complemented with a local characterization of the discontinuous deformations in the small scales. In particular, we refer to the analyses presented in [16] of the resulting finite element approximations of the discontinuous solutions, including the identification of the different effective length scales appearing in the computed solutions. We follow this approach in the developments presented here.

Alternative treatments of these considerations include the extended finite element method (often referred to as X-FEM), as developed in [17–24], among others. Rather than considering the aforementioned local element-based enhancements, this approach is characterized by the incorporation of additional degrees of freedom through nodal enrichments based on the concept of partition of unity. We refer to [25] for comparisons between the two different finite element formulations (E-FEM vs. X-FEM) for the inclusion of strong discontinuities, especially in terms of computational cost since the full locality and the aforementioned elimination of the added degrees of freedom associated to the propagating discontinuity surface is lost in the X-FEM approach.

Early attempts in the formulation of finite elements with embedded strong discontinuities, the focus here, considered piece-wise constant approximations of the displacement jumps in combination with linear triangles [9,11,12,14] or quadrilateral elements [26]. But the consideration of only constant jumps was observed in [27] to lead to difficulties in the modeling of softening hinges in plates as strong discontinuities in the deflection and rotation fields characterizing the kinematics of these structural members. The coupled nature of these generalized displacement fields clearly requires the consideration of, say, linear deflection jumps in combination with constant rotation jumps to accomplish the proper kinematic characterization of these solutions, and hence avoid over-stiff solutions (a response usually refer to as stress-locking, as discussed below). Crucial to the design of the finite element enhancements, an