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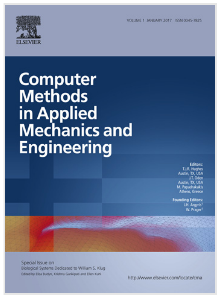

Optimal reduction of numerical dispersion for wave propagation problems. Part 1: Application to 1-D isogeometric elements

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Optimal Reduction of Numerical Dispersion for Wave Propagation Problems. Part 1: Application to 1-D Isogeometric Elements.

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Abstract

A numerical technique with the optimal coefficients of the stencil equation has been suggested. Based on this approach, new high-order isogeometric elements with the reduced dispersion error have been developed for wave propagation problems in the 1-D case. By the modification of the mass and stiffness matrices, the order of the dispersion error is improved from order $2p$ (the conventional elements) to order $4p$ (the new elements) where p is the order of the polynomial approximations. It was shown that the new approach yields the maximum order of the dispersion error for the stencil equations related to the high-order isogeometric elements. The analysis of the dispersion error of the high-order isogeometric elements with the lumped mass matrix has also shown that independent of the procedures for the calculation of the lumped mass matrix, the second order of the dispersion error cannot be improved with the conventional stiffness matrix. However, the dispersion error for the lumped mass matrix can be improved from the second order to order $2p$ by the modification of the stiffness matrix. The numerical examples confirm the computational efficiency of the new high-order isogeometric elements with reduced dispersion. We have also showed that numerical results obtained by the new and conventional high-order isogeometric elements may include spurious oscillations due to the dispersion error. These oscillations can be quantified and filtered by the two-stage time-integration technique recently developed in our papers. The approach developed in the paper can be directly applied to other space-discretization techniques with similar stencil equations.

Keywords: isogeometric elements, high-order elements with reduced dispersion, wave propagation, numerical dispersion

In the paper the reduction of the numerical dispersion error for wave propagation problems with the application to isogeometric elements in the 1-D case (Part 1) and the 2-D case (Part 2; see [1]) is considered. Wave propagation in an isotropic homogeneous medium is described by the following scalar wave equation in domain Ω:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \nabla^2 u = 0\,, \tag{1}$$

with the boundary conditions $\boldsymbol{n} \cdot \nabla u = g_1$ on Γ^t and $u = g_2$ on Γ^u, and the initial conditions $u(\boldsymbol{x}, t = 0) = g_3$, $v(\boldsymbol{x}, t = 0) = g_4$ in Ω. Here, u is the field variable, $v = \dot{u}$ is the velocity, c is the wave velocity, t is the time, Γ^t and Γ^u denote the natural and essential boundaries, g_i $(i = 1, 2, 3, 4)$ are the given functions, $\boldsymbol{n}$ is the outward unit normal on Γ^t. The application of the continuous Galerkin approach and the space discretization (e.g., the finite elements, spectral elements, isogeometric elements; see [2, 3, 4] and others) to Eq. (1) leads to a

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