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Multi-scale modal analysis for axisymmetric and spherical symmetric structures with periodic configurations

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Highlights

- Multi-scale expansions of the eigenfunctions and eigenvalues are formulated.
- The effective materials manifest anisotropic behavior with different properties in circumferential direction.
- Analytical solutions of homogenization and cell problems are derived for the layered structure.
- Finite element procedures are established for the asymptotic analysis.
- Numerical examples demonstrate the effectiveness of our proposed models.

Abstract

A new modal analysis method with second-order two-scale (SOTS) asymptotic expansion is presented for axisymmetric and spherical symmetric structures. The symmetric structures considered are periodically distributed with homogeneous and isotropic constituent materials. By the asymptotic expansion of the eigenfunctions, the homogenized modal equations, the effective materials coefficients, the first- and second-order correctors are obtained. The derived homogenized constitutive relationships are the same as the ones which serve to homogenize the corresponding static problems. The eigenvalues are also expanded to the second-order terms and using the so called "corrector equation", the correctors of the eigenvalues are expressed in terms of the first- and second-order correctors of the eigenfunctions. The anisotropic materials are obtained by homogenization with different properties in the circumferential direction. Especially for the two-dimensional axisymmetric layered structure, the one-dimensional plane axisymmetric and spherical symmetric structures, the homogenized eigenfunctions and eigenvalues, as well as their corresponding correctors are all solved analytically. The finite element algorithm is established, three typical numerical experiments are carried out and the necessity of the second-order correctors is discussed. Based on the numerical results, it is validated that the SOTS asymptotic expansion homogenization method is effective to identify the eigenvalues of the axisymmetric and spherical symmetric

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structures with periodic configurations and the original eigenfunctions with periodic oscillation can be reproduced by adding the correctors to the homogenized eigenfunctions.

Keywords: Asymptotic expansion homogenization method; SOTS finite element algorithm; Modal analysis; Eigenvalue problem; Axisymmetric and spherical symmetric structure; Periodic configuration

1. Introduction

Modal analysis is important to study the stability of a mechanical structure, which is routinely performed in industry. It involves the computation of the eigenvalues and the eigenmodes (eigenfunctions) for the vector displacement field [1] and is typically done during the design process to identify resonance frequencies (i.e. eigenvalues). In fact, resonance vibrations can cause the structure to wear out unreasonably fast or even fail due to fatigue [2,3]. With the widespread emergence of new composite materials, it is necessary to study the spectral properties of strongly heterogeneous materials.

Mathematically, the modal analysis essentially involves the general eigenvalue problems [4] and can be solved numerically by the iterative algorithms in [5]. However, almost all the composite materials have multiscale features in nature [6], i.e. the scale of structure is much larger than the feature size of materials or constituents. When considering the structures composed of heterogeneous materials, the computational amounts are so large that we have to choose the grid size small enough to capture the local fluctuations of the solutions and simulate the macroscopic mechanical properties of these composites as the element size should be smaller than the minimum size of materials. To simplify the analysis and save the computational memory, the Asymptotic Expansion Homogenized (AEH) method is introduced in [7–11], and the governing equations with rapidly oscillating coefficients are reduced as the equations for media with homogenized or effective coefficients, which are often constant or continuous throughout the structure. Following the idea of the multi-scale expansion, various physical and mechanical problems have been brought into the framework, many homogenized models are obtained; moreover, by proper correctors, the local fluctuation can be reproduced effectively. Some featured researches have involved the conduction–radiation heat transfer problem [12], the coupled thermo-mechanical problem [13], the large deformation problem [14], the elastic and thermo-elastic problem in cylindrical coordinates [15,16] and the nonlinear coupled thermoelectric problem [17,18]. Besides the classical finite element computation of the asymptotic models, the Bloch wave method has been developed [19,20] to provide a better approximation. Based on this research, a Second-Order Two-Scale (SOTS) method is developed [21] to predict the physical and mechanical performance of the composite materials more accurately by considering the second-order corrections, which is applied to the finite element computation of various heat transfer and elastic deformation problems. It is verified [22] that the second-order correctors in the asymptotic expansion play an important role to capture local oscillation of the corresponding physical fields. Then, the SOTS method is extended to different kinds of thermal and mechanical problems and various second-order correctors are developed, including the coupled thermoelastic problem [23], the quasi-periodic problem [24], the conduction–radiation coupled heat transfer problem [25], the static elastic problem and the dynamic thermal–mechanical coupling problem in the axisymmetric and spherical symmetric structures [26,27]. Recently, the SOTS expansion for the heat conduction problem is discussed [28] in a general curvilinear coordinates.

For the eigenvalue problems, Oleinik et al. [9] and Jikov et al. [10] investigated the Sturm–Liouville equation with oscillating coefficients and obtained the one-dimensional asymptotic expansion of eigenvalue and eigenfunctions. Kesavan [29] studied the homogenization of eigenvalue problems for a second-order elliptic operator with rapidly oscillating coefficients, in which the "corrector equation" is introduced to obtain the expansion of the eigenvalues and numerical experiments are performed [30] by considering the second-order correctors. Vanninathan [31] studied the homogenization of eigenvalue problems in perforated domains with Dirichlet, Stekloff and Neumann types. The two-scale convergence of Stekloff eigenvalue problems in perforated domains was discussed by Douanla [32]. Cao and Cui [33] discussed the asymptotic expansions of the eigenvalues and eigenfunctions of the Dirichlet problem for the second-order elliptic equations in general and perforated domain. For a fixed domain, a kind of Steklov eigenvalue problem was presented and the finite-element computation was performed in Cao et al. [34]. The one-dimensional eigenvalue problem in a periodic medium with an interface was studied by Allaire and Capdeboscq [35].