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An Energetically Consistent Concurrent Multiscale Method For Heterogeneous Heat Transfer and Phase Transition Applications

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Abstract

A concurrent multiscale method is developed to model time-dependent heat transfer and phase transitions in heterogeneous media and is formulated in a way such that the energy being exchanged between scales is conserved. Ensuring this energetic consistency among scales enables the implementation of high fidelity physics-based models at critical locations within the coarse-scale to temporally and spatially resolve highly complex and localized phenomena. To achieve this, only Neumann boundary conditions are applied over the fine scale domain, ensuring a conservative formulation. The coarse-scale solution is used to reconstruct these Neumann boundary conditions on the fine scale, which are then used to evolve a separate system of governing equations. The results on the fine scale are then sent back to the coarse scale through an energy-based homogenization scheme. Transient simulations for the heat equation are implemented with the proposed method to demonstrate its accuracy in energy conservation and effectiveness, including the coupling of a phase field model at the fine scale to a coarse-scale heat equation.

Keywords: Concurrent multiscale method, Multilevel finite elements, Heat equation, Phase field models

1. Introduction

Heat transfer mechanisms have been extensively studied and manipulated to drive structural changes of raw materials in manufacturing applications. These structural changes are achieved through the use of extreme cooling rates and thermal gradients to induce complex phase transitions such as melting solidification. Phase transitions extend across multiple scales: the solidification front can exhibit nonlinear dynamic behavior on the fine scale to absorb or release latent heat and alter the total energy of the coarse scale system. Computational modeling can play a pivotal role in elucidating the governing physics that drive these phase transitions; however, the inherent multiscale nature of these systems requires careful consideration of the model being applied such that these important dynamic physical features are spatially and temporally resolved.

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