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An efficient Monte Carlo interior penalty discontinuous Galerkin method for elastic wave scattering in random media

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Abstract

This paper develops and analyzes an efficient Monte Carlo interior penalty discontinuous Galerkin (MCIP-DG) method for elastic wave scattering in random media. The method is constructed based on a multi-modes expansion of the solution of the governing random partial differential equations. It is proved that the mode functions satisfy a three-term recurrence system of partial differential equations (PDEs) which are nearly deterministic in the sense that the randomness only appears in the right-hand side source terms, not in the coefficients of the PDEs. Moreover, the same differential operator applies to all mode functions. A proven unconditionally stable and optimally convergent IP-DG method is used to discretize the deterministic PDE operator, an efficient numerical algorithm is proposed based on combining the Monte Carlo method and the IP-DG method with the LU direct linear solver. It is shown that the algorithm converges optimally with respect to both the mesh size h and the sampling number M, and practically its total computational complexity only amounts to solving a few deterministic elastic Helmholtz equations using a Gaussian elimination direct linear solver. Numerical experiments are also presented to demonstrate the performance and key features of the proposed MCIP-DG method.

Keywords: Elastic Helmholtz equations, random media, Rellich identity, discontinuous Galerkin methods, error estimates, Monte Carlo method

1. Introduction

Elastic wave scattering problems arise from applications in a variety of fields including geoscience, image science, the petroleum industry, and the defense industry, to name a few. Such problems have been extensively studied both analytically and numerically in the past several decades (cf. [18, 21] and the

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