A 3-field formulation for strongly transversely isotropic compressible finite hyperelasticity

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Abstract

A Hu-Washizu type 3-field virtual work principle for strongly transversely isotropic compressible finite hyperelasticity is developed, implemented and verified. The independent variables are: the stretch, its energy conjugate uniaxial tension and the displacement. The formulation is based on a new decomposition of the deformation gradient into a simply stretchless part and a lateral contraction-free uniaxial extension. In the fully constrained limit the formulation provides the constraint manifold setting of hyperelasticity with the simple internal kinematic constraint of inextensibility. The finite element formulation is implemented in an *hp*-adaptive code providing the proper flexible environment for finite elements with variable order and mixed interpolation. A study using a semi-inverse analytical solution corroborates the convergence characteristics for *h*-refinements and *p*-enrichments of the 3-field implementation. The case of isostatic loading of a circular cylinder reinforced with a single family of fibres, and the pressurisation of a saccular aneurysm-like configuration with circumferential fibre reinforcement are used for verification and illustration. *A posteriori* residual discretisation error estimation is used for making mesh refinements in the latter case. Typical applications are found in soft tissue biomechanics.

Keywords: compressible, transversely isotropic, hyperelasticity, finite element, *hp*-adaptivity

1. Introduction

A suitable computational weak formulation for *compressible but possibly inextensible* transversely isotropic finite hyperelasticity is developed herein. The request for such a finite element based approach is found, for example, in soft fibre reinforced rubber-like composites, and in the biomechanics of soft tissue. Our attention is drawn to the prevalent *measurable finite compressibility* recently pointed out, see for example [1, 2, 3]. It concerns in particular the mechanics of arteries.

With rising blood pressure the collagen fibre reinforcement of an artery provides an exponential fibre stiffening [4, 5]. With a compressible material [1] it quickly out-grows the low stiffness in shear characterising the rubber-like ground substance material. The tentative computational range ($E_f/\mu \geq 1000$) and scenario of interest related to artery mechanics is depicted in Figure 1.

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