A level-set-based topology optimisation for acoustic–elastic coupled problems with a fast BEM–FEM solver

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Abstract

This paper presents a structural optimisation method in three-dimensional acoustic–elastic coupled problems. The proposed optimisation method finds an optimal allocation of elastic materials which reduces the sound level on some fixed observation points. In the process of the optimisation, configuration of the elastic materials is expressed with a level set function, and the distribution of the level set function is iteratively updated with the help of the topological derivative. The topological derivative is associated with state and adjoint variables which are the solutions of the acoustic–elastic coupled problems. In this paper, the acoustic–elastic coupled problems are solved by a BEM–FEM coupled solver, in which the fast multipole method (FMM) and a multi-frontal solver for sparse matrices are efficiently combined. Along with the detailed formulations for the topological derivative and the BEM–FEM coupled solver, we present some numerical examples of optimal designs of elastic sound scatterer to manipulate sound waves, from which we confirm the effectiveness of the present method.
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1. Introduction

Computer simulations play an important role in modern product manufacturing process in various engineering fields. The concept of computer aided engineering (CAE) is now widely accepted in some industries, which utilises numerical analysis to aid in tasks to evaluate the performance of engineering products. With the help of CAE, the total cost and period for product developments have considerably been reduced. In these days, use of computer simulations is not limited to performance evaluation, but is extended to design process. As such an attempt, we can mention a structural optimisation, which is classified into sizing, shape and topology optimisation [1]. The topology optimisation is considered as the most powerful design method in structural optimisations since it can design not only the shape

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but also the topology of devices, i.e., the topology optimisation allows a nucleation of a new material and/or hole in its process. Hence, the obtained optimal design by the topology optimisation is less affected by an initial guess than the other structural optimisations.

After a pioneering work in [1], the topology optimisation is intensively studied mainly in the field of structural mechanics in order to design light but stiff structural members [1–3]. Recently, one of the main interests in the topology optimisation community is to widen the applicability of the topology optimisation to problems in various engineering fields other than structural mechanics, such as thermal problems [4–6], fluid problems [7–9], elastodynamic problems [10–13], electromagnetic wave problems [14–18], and so on. There are also many efforts to extend the topology optimisation into various design problems in acoustics such as an acoustic horn to maximise sound level [19], an acoustic metamaterial which realises a material with negative effective bulk modulus [20], and a poroelastic sound-proofing material [21–25]. We think, however, the applicability of these existing methods for industrial designs is still limited because these topology optimisations use the finite element method (FEM) to solve boundary value problems involved in sensitivity analysis. Since the acoustic problem is often defined in an unbounded domain, the unbounded domain is approximated with a large one in FEM, which leads to an unexpectedly large scale problem. Especially, this problem may be crucial when sound sources and/or observation points are located far away from the design object because, for FEM, both sources and observation points as well as the design objects must be covered by finite element mesh. Further, an artificial boundary condition such as perfect matched layer (PML) is required to make sure that the scattered wave does not reflect on the truncated boundary.

On the other hand, when the boundary element method (BEM) is employed to solve wave scattering problems, only the boundary of the domain is needed to be discretised, which considerably reduces the number of elements. Even when sources and/or observation points are far away from the design objects, the boundary element mesh is required only on the boundary. Also, the scattered fields by the BEM automatically satisfy the radiation condition, i.e., no artificial boundary condition is required to deal with the unbounded domain with the BEM. Thus, the BEM is more suitable for topology optimisations in wave problems than the FEM. As pioneering works on BEM-based topology optimisations, we can mention Abe et al. [26] and Du and Olhoff [27], In the first one, they have solved a two-dimensional shape optimisation problem by the BEM to design a sound barrier. In the second one, they have solved a topology optimisation problem for a noise reduction device from vibrating structures, in which they use an approximated boundary integral formulation for high frequency problems. The applicability of these BEM-based method is limited to either two-dimensional problem [26] or high frequency problem [27] because a naive BEM for three-dimensional realistic scale problems is too expensive. It is inevitable to accelerate the BEM by, for example, fast multipole method (FMM) [28,29], $\mathcal{H}$-matrix algebra [30] and fast direct solver [31] for topology optimisation in sound problems.

In order to realise a topology optimisation for three-dimensional realistic design problems of wave devices, we have been investigating level-set-based topology optimisations with the BEM accelerated by the FMM. In our methodology, a candidate for optimal configuration is expressed with a level set function which is iteratively updated with the help of the topological derivative [32–35] to find an optimal distribution of scatterers. Since we have adopted the level set method and BEM, our methods are greyscale free, i.e. the boundary of the design object is clearly expressed. In our methods, the boundary elements are re-meshed from the distribution of the level set function at the every step of the optimisation. In [32], we have investigated a topology optimisation for rigid materials to minimise sound pressure on some observation points. We have extended the methodology to find an optimal allocation of sound absorbers in [36], in which a sound absorbing material is modelled with the impedance boundary condition. The impedance boundary condition is, however, not appropriate to model sound absorbing material in some applications. For example, in analysis with the impedance boundary condition, penetrated sound waves in the sound absorbing materials cannot explicitly be observed, and vibrations in the sound absorber itself are neglected.

In this study, to further enhance the applicability of our methodology, we present a level-set-based topology optimisation in acoustic–elastic coupled problems, with which the vibrations of sound scatterers made of elastic materials are explicitly evaluated. In order to solve the acoustic–elastic coupled problem, we adopt a BEM–FEM solver which solves the acoustic and elastic field by BEM and FEM, respectively. This choice is reasonable since, with our settings, the elastic material is in a bounded domain while the acoustic host matrix is an unbounded domain. Although the acoustic–elastic coupled problem can appropriately be solved by the BEM [37,38], we use the BEM–FEM solver [39] since the solver can naturally be extended to deal with elastic material other than the isotropic one such as anisotropic material and Biot's poroelastic material [40,41]. So far, some fast techniques [42]