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Spectral element method for three dimensional elliptic problems with smooth interfaces☆

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Dedicated to Professor J. Tinsley Oden on the occasion of his 80th birthday

Highlights

- A fully non-conforming LSSEM is studied for 3D elliptic interface problems.
- Differentiability estimates and the main stability theorem are proven.
- Efficient diagonal preconditioner is discussed for 3D elliptic interface problems.
- Exponential convergence rate is shown through error estimate and numerical examples.
- Numerical results are obtained for both straight and curved interfaces.

Abstract

In this paper we propose a least-squares spectral element method for three dimensional elliptic interface problems. The differentiability estimates and the main stability theorem, using non-conforming spectral element functions, are proven. The proposed method is free from any kind of first order reformulation. A suitable preconditioner is constructed with help of the regularity estimate and proposed stability estimates which is used to control the condition number. We show that these preconditioners are spectrally equivalent to the quadratic forms by which we approximate them. We obtain the error estimates

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which show the exponential accuracy of the method. Numerical results are obtained for both straight and curved interfaces to show the efficiency of the proposed method.

Keywords: Least-squares methods; Non-conforming spectral element method; Linear elliptic PDE in three dimensions; Interfaces; Preconditioner; Exponential accuracy

1. Introduction

In the areas of applied mathematics, engineering and scientific computing, many problems attract much attention to compute the numerical solution of elliptic problems which have discontinuous coefficients and singular sources [1–25]. These problems have great impact in the field of fluid dynamics [26–30], electro-magnetics [31–33], material science [34,35] and biologic systems [36–39]. Many numerical methods such as finite difference methods, finite element methods and spectral method may not work properly due to the lack of regularity in the solution. In some worst cases, it may diverge too. To construct a very efficient and accurate numerical method for this class of problems is a challenge.

Fedkiw, Osher and their coworkers [40,41] proposed a Ghost Fluid Method (GFM) for the treatment of discontinuities and interfaces. GFM is used to sharp interface. Cai et al. [42] presented an upwinding embedded boundary method for dielectric media. Oevermann et al. [43] discussed a second-order finite volume based method using bilinear ansatz functions on Cartesian grids and compact stencils. The other different approach for embedding complex geometry is integral equation method [44–46]. The most popular approach the Immersed Interface Method (IIM) for interface problems was given by Leveque and Li [19]. Some higher order interface methods are proposed in [47,45,48–51]. In general, these high order methods are not suitable for those problems which have both material interfaces and high frequency waves. In such cases conventional local adaptive refinement approaches do not work well. Recently, Wei and their coworkers presented higher order finite-difference time-domain (FDTD) methods via derivative matching for Maxwell's equations with material interfaces to solve electromagnetic wave propagation and scattering in dielectric media [49]. They also used this idea to solve the fourth-order beam equation with free boundary conditions [52] and generalized the earlier matched interface and boundary (MIB) method for solving elliptic equations with curved interfaces [50,51]. In [53,39], Wei and their coworkers proposed a three-dimensional (3D) MIB scheme for treating geometric singularities.

The geometric singularities (such as sharp-edged, sharp-wedged and sharp-tipped interfaces) arise on elliptic equations with smooth interfaces. In a number of science and engineering applications, there are a class of problems for example wave-guides analysis [54], electromagnetic wave scattering and propagation [55–57], friction modeling [58], plasma–surface interaction [59] and turbulent-flow [60] having geometric singularities. In presence of geometric singularities (e.g. cusps and self-intersecting surfaces), it is difficult to prove the accuracy, convergence and stability of the numerical method of the Poisson–Boltzmann equation for the electrostatic analysis of biomolecules [36]. Moreover, an efficient high order numerical method for this class of problems is not easy to develop because of the lack of regularity at the interface. Apart from this, interface problems do not have well defined gradient near the tip of a sharp-edged interface.

To solve the interface problems, the large class of existing methods are conforming finite element methods which require the triangulations in different subregions to be matching on the interface. Conforming methods may pose serious restrictions when the physical solutions of the interface problems are of different scales in different subregions. The good alternatives to relax such restrictions are nonconforming methods like mortar finite element method and discontinuous Galerkin methods. In [16], Huang et al. presented mortar finite element method for elliptic interface problems and proved the optimal L^2 and H^1 error estimates (when the interface is of arbitrary shape but smooth) even though the regularity of the true solution is low. In [5], Lamichhane et al. also discussed the mortar finite element method and dual Lagrange multipliers for the interface problems. In [15], Hansbo et al. proposed an unfitted finite element method, based on Nitsche's method, for elliptic interface problems. In [18], Xia et al. presented a Galerkin formulation of the MIB method for three dimensional elliptic interface problems.

In the recent years, spectral methods are extensively used for solving the partial differential equations because of high order of accuracy, see [61–66] and the references therein. In particular, least-squares spectral element