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High Order Boundary and Finite Elements for 3D Fracture Propagation in Brittle Materials

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Abstract

The quasi-static propagation of fracture in brittle materials was studied in several recent publications. A variational formulation [1, 2, 3] led to three-dimensional crack tracking strategies [4, 5, 6]. One of the complexities of this new class of algorithms is the evaluation of high-order terms of the expansion of the crack opening and sliding. In this paper, new types of finite and boundary elements are formulated that capture the near crack-front asymptotical displacement behavior up to the order 3/2. The use of these elements with the quasi-static crack propagation algorithms of the above references is demonstrated for a simple crack configuration.

1 Introduction

A recent series of papers [3, 4, 5, 6] by two of the current authors is concerned with the formulation of three dimensional crack tracking algorithms assuming linear elastic fracture mechanics (LEFM). In these papers it is shown that the evolution of crack shapes is dependent on the second order term in the expansion of the crack opening displacement in the vicinity of the crack front. While there has been much effort devoted to numerically modeling the theoretically predicted singular near-front singular fields [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18], relatively little attention has been devoted to numerically modeling this second order term. This paper is concerned with boundary and finite element estimations of this term.

The evolution of cracks in continua can be thought of as having three regimes: loading without crack growth, stable, and unstable crack propagation. The global *incremental* quasi-static fracture propagation problem seeks an expression of the *crack growth rate* at all points along the crack front due to changes in applied loads or displacements for the first two phases of a fracturing process. Provided that the state of stress and the history of crack propagation at a given time t is known, in the present note the crack front propagation rate due to an increase in magnitude of the external actions is expressed as a function of the stress and of the history.

Several scientific publications put Linear Elastic Fracture Mechanics (LEFM) in analogy with the thermodynamics of standard dissipative processes [19, 20, 21]. The global incremental quasi-static fracture propagation problem in brittle materials has been investigated recently in this framework, motivated by an analogy between rigid-plasticity and LEFM. A brief account of the theory is provided in section 3. Asymptotic expansions for stress intensity factors (SIFs) in three dimensions play an important role in the theory. They have been studied in [22] as well as in [23] and Leblond's notation will be used in what follows. In [2] and [3] a variational formulation for the global incremental quasi-static propagation was given based on a Colonnetti [24] decomposition interpretation of the asymptotic expansions. Minimum theorems for