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A *J-integral*-based arc-length solver for brittle and ductile crack propagation in hyperelastic solids

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Abstract

Arc-length methods based on Newton-Rahpson iterative numerical solvers are indispensable in non-linear mechanics to trace the equilibrium curves of systems with snap-back or snap-through behaviours. An example is a crack propagating in a rubber-like solid, which can generate extremely sharp snap-backs when the elastic energy is released.

A standard arc-length solver satisfies the arc-length constraint which only limits the increment in displacements and in load level to a fixed amount. Thus, it cannot guarantee the satisfaction of the energy balance in the rate form. This work proposes an arc-length based on the J-integral, and assumes the Griffith balance as a constraint equation.

Firstly, a fracture criterion (critical load and direction of crack increment) is formulated, consistent with the thermodynamic principle of maximum dissipation: a crack propagates in the direction of the maximum strain energy release rate. For this purpose, this paper provides an explicit and simple expression of the *J-integral* for varying angles. This fracture criterion does not require the computation of mixed mode Stress Intensity Factors, or asymptotic solutions or derivatives of the tangent stiffness matrix. By doing so, it is shown that the proposed method easily reconciles with the theory of Configurational Forces.

Secondly, because of the explicit expression of *J* provided in this paper, it is easy to linearize the discretized equations of motions consistently.

The method proves able to handle very sudden snap-backs occurring under large strains, and both brittle and ductile crack propagation. A particular example of such behaviour are *kirigami* (paper-cuts) structures made of graphene. The proposed arc-length can capture, for these structures, the transitions from brittle into ductile crack propagation for the same crack patterns, but different pre-crack lengths.

Keywords:
arc-length, fracture, hyperelastic, numerical continuation, J-integral, strain energy release rate, snap-back, graphene, kirigami

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