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Isogeometric analysis with geometrically continuous functions on planar multi-patch geometries

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Highlights

- Construction of a basis for bicubic and biquartic C^1-smooth isogeometric functions on planar bilinear multi-patch domains.
- The basis functions are described by simple explicit formulas for their spline coefficients.
- Numerical experiments (e.g. solving the biharmonic equation) showed optimal rates of convergence.

Abstract

We generate a basis of the space of bicubic and biquartic C^1-smooth geometrically continuous isogeometric functions on bilinear multi-patch domains $\Omega \subset \mathbb{R}^2$. The basis functions are obtained by suitably combining C^1-smooth geometrically continuous isogeometric functions on bilinearly parameterized two-patch domains (cf. [18]). They are described by simple explicit formulas for their spline coefficients.

These C^1-smooth isogeometric functions possess potential for applications in isogeometric analysis, which is demonstrated by several examples (such as the biharmonic equation). In particular, the numerical results indicate optimal approximation power.

Keywords: Isogeometric analysis; C^1-smooth isogeometric functions; Geometrically continuous isogeometric functions; Multi-patch domain; Biharmonic equation

1. Introduction

Isogeometric Analysis (IgA) is a promising framework for performing numerical simulation, which uses the same (rational) spline function space for representing the geometry of the physical domain and describing the solution space [1,2]. Multi-patch parameterizations have been introduced in order to perform isogeometric simulations on more complex geometries. Two main approaches for coupling the individual patches exist.

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The first one does not modify the isogeometric spaces on the individual patches but uses other techniques to achieve global smoothness of the solution (at least approximately). These include the discontinuous Galerkin method [3,4], the use of Nitsche's technique [5,6], the mortar approach [7–9] and domain decomposition methods [10,11]. Typically, these techniques aim at ensuring C^0-continuity weakly of the resulting numerical solution.

The second approach uses a globally defined basis for the isogeometric simulation on the multi-patch domain, thereby modifying the spaces on the individual patches and coupling them explicitly. The case of C^0-smoothness is well understood: The notion of isogeometric spline forests was introduced in [12] and was extended recently by enhancing the smoothness across interfaces and introducing hierarchical spline refinement [13]. Isogeometric function spaces possessing higher regularity, however, are more difficult to construct and require the classical notion of geometric continuity, see [14] and the references therein.

Geometric continuity is a well-established approach in Computer Aided Geometric Design for designing smooth multi-patch surfaces possessing extraordinary vertices (EVs) [15,16], i.e., surfaces composed of quadrilateral patches where other than 4 patches may meet in some vertices. The construction of C^s-smooth isogeometric functions on multi-patch domains is based on the observation – which has been formalized firstly by Groisser and Peters [17] – that the C^s-smoothness of an isogeometric function is equivalent to the geometric smoothness of order s (G^s-smoothness) of its graph surface,[1] where s is a positive integer. Motivated by this we denote the C^s-smooth functions on a multi-patch domain as *C^s-smooth geometrically continuous isogeometric functions* [18].

We restrict ourselves to the case $s = 1$. Two different strategies following the concept of geometric smoothness have been explored.

The first one derives C^1-smooth geometrically continuous isogeometric functions from existing constructions for G^1-smooth multi-patch surfaces that originated in geometric design. Related results include the recent publications [19–21], which are based on different G^1-smooth multi-patch spline surfaces, and the use of EVs in T-spline-based representations [22]. Numerical results indicate that the accuracy of the results may deteriorate in the vicinity of the EVs. Moreover, the construction of nested isogeometric spaces via h-refinement remains an open problem if EVs are present.

The second strategy employs a basis of the entire space of C^1-smooth functions on a particular class of multi-patch geometries, cf. [18,23], and uses it to describe the geometry and to perform isogeometric simulations. A first step was presented in [18], where we analyzed the spaces of bicubic and biquartic C^1-smooth geometrically continuous isogeometric functions on bilinearly parameterized two-patch domains. Furthermore we developed a simple framework for the construction of a basis in the general setting and obtained promising numerical results indicating optimal approximation power. These are also supported by the recent results in [24].

The approaches [18,23] (and also the present approach) are based on *assembling* B-spline isogeometric patches. It is different from the classic higher order Finite Elements Methods (FEM) such as Clough–Tocher macro triangles and their generalization to bivariate triangular splines, cf. [25]. These FEM based approaches construct C^1-smooth bases functions per element. In contrast, the present and the earlier approaches [18,23] deal only with the patches' interfaces and therefore the interior of each patch has the regularity of the corresponding isogeometric B-Spline (which can be higher than C^1, especially for higher order splines). Moreover, the patches can have vertices of any valence, that would not be possible with classical bivariate Hermite type constructions like Bogner–Fox–Schmidt quadrilaterals, cf [26].

The present work extends the earlier results from [18], obtained for bilinearly parameterized two-patch domains, to bilinearly parameterized multi-patch domains. This generalization increases the geometric flexibility of the construction, while we also present numerical results indicating that the optimal approximation properties are preserved. We describe the construction of bicubic and biquartic C^1-smooth geometrically continuous isogeometric basis functions. The constructed basis allows the use of the same function space for performing simulation and describing the geometry in agreement with the main ideas of IgA. The basis functions are specified by simple explicit formulas for their spline coefficients in contrast to [18], where Bézier coefficients are used to present a basis of bi-degree $(4, 4)$.

The main differences and novelties of our work compared to [23] are as follows: While our construction is based on bicubic and biquartic C^1-smooth geometrically continuous isogeometric basis functions, the work [23] mostly deals with biquintic functions and the biquartic case is only described for a specific setting of the multi-patch domain. In particular, the construction of bicubic C^1-smooth geometrically continuous isogeometric functions has not been

[1] The graph surface of an isogeometric function is the 3D surface where the first two coordinates are the coordinates of the physical domain and the third coordinate is the associated value of the isogeometric function, compare Eq. (10).

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