The value of continuity: Refined Isogeometric Analysis and fast direct solvers

Daniel Garcia, David Pardo, Lisandro Dalcin, Maciej Paszyński, Nathan Collier, Victor M. Calo

PII: S0045-7825(16)30958-6
DOI: http://dx.doi.org/10.1016/j.cma.2016.08.017
Reference: CMA 11098

To appear in: *Comput. Methods Appl. Mech. Engrg.*

The Value of Continuity: Refined Isogeometric Analysis and Fast Direct Solvers

Daniel Garcia[a,*], David Pardo[b,a,c], Lisandro Dalcin[d,e,f], Maciej Paszyński[g], Nathan Collier[h], Victor M. Calo[i,j]

[a] *Basque Center for Applied Mathematics, (BCAM), Bilbao, Spain.*
[b] *Department of Applied Mathematics, Statistics, and Operational Research, University of the Basque Country UPV/EHU, Leioa, Spain*
[c] *Ikerbasque (Basque Foundation for Sciences), Bilbao, Spain.*
[d] *Center for Numerical Porous Media, King Abdullah University of Science and Technology (KAUST), Thuwal, Saudi Arabia.*
[e] *Consejo Nacional de Investigaciones Científicas y Técnicas, Santa Fe, Argentina.*
[f] *Universidad Nacional del Litoral, Santa Fe, Argentina.*
[g] *Department of Computer Science, AGH University of Science and Technology, Krakow, Poland.*
[h] *Climate Change Science Institute, Oak Ridge National Laboratory, Oak Ridge, Unites States.*
[i] *Applied Geology, Western Australian School of Mines, Curtin University, Bentley, WA, Australia.*
[j] *Mineral Resources, Commonwealth Scientific and Industrial Research Organisation (CSIRO), Kensington, WA, Australia.*

Abstract

We propose the use of highly continuous finite element spaces interconnected with low continuity hyperplanes to maximize the performance of direct solvers. Starting from a highly continuous Isogeometric Analysis (IGA) discretization, we introduce C^0-separators to reduce the interconnection between degrees of freedom in the mesh. By doing so, both the solution time and best approximation errors are simultaneously improved. We call the resulting method "refined Isogeometric Analysis (rIGA)". To illustrate the impact of the continuity reduction, we analyze the number of Floating Point Operations (FLOPs), computational times, and memory required to solve the linear system obtained by discretizing the Laplace problem with structured meshes and uniform polynomial orders. Theoretical estimates demonstrate that an optimal continuity reduction may decrease the total computational time by a factor between p^2 and p^3, with p being the polynomial order of the discretization. Numerical results indicate that our proposed rIGA method delivers a speed-up factor proportional to p^2. In a $2D$ mesh with four million elements and $p = 5$, the linear system resulting from rIGA is solved 22 times faster than the one from highly continuous IGA. In a $3D$ mesh with one million elements and $p = 3$, the linear rIGA system is solved 15 times faster than the IGA one.

Keywords: Isogeometric Analysis (IGA), Finite Element Analysis (FEA), refined Isogeometric Analysis (rIGA), Direct solvers, Multi-frontal solvers, k-refinement.

1. Introduction

In order to solve numerically a problem governed by partial differential equations (PDEs) and specific boundary conditions (BCs), we often resolve a system of algebraic equations which conforms the discrete representation of the problem. Discretization approaches such as finite element analysis (FEA) or isogeometric analysis (IGA) are frequently employed to generate this algebraic system. These methods discretize the governing PDEs by using a variational formulation and trial and test functions defined by their respective basis functions [1, 2].

In traditional FEA, the basis functions are defined on a reference element, and a mapping to the physical element is employed [1]. Isogeometric Analysis (IGA) seeks to eliminate the need for an interface lying between the geometrical representation built with Computed-Aided Design (CAD) and the analysis performed with FEA, since basis functions are defined themselves using conventional CAD functions. Although many obstacles still remain in the present-day CAD software, this feature avoids the need to define a secondary set of functions (and the corresponding transfer operators) for the numerical analysis [2].

*Corresponding author
Email address: dgarcia@bcamath.org (Daniel Garcia)
URL: www.bcamath.org/dgarcia (Daniel Garcia)