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Quadrilateral and hexahedral mesh generation based on surface foliation theory

Na Lei[a,d], Xiaopeng Zheng[c], Jian Jiang[b], Yu-Yao Lin[b], David Xianfeng Gu[b,d,*]

[a] *School of Software and Technology, Dalian University of Technology, Dalian, China*
[b] *Computer Science Department, Stony Brook University, New York, United States*
[c] *School of Mathematics, Jilin University, Changchun, China*
[d] *Beijing Advanced Innovation Center for Imaging Technology, Capital Normal University, Beijing, China*

Highlights

- Rigorous: Lay down a solid theoretic foundation for structured hex-meshing.
- Automatic: The algorithm pipeline can be fully automatic with minimal user input.
- Regular: The hex-meshes are with minimal singular lines.
- Conformal: The quad-meshes are conformal to the geometry of the input model.

Abstract

For the purpose of isogeometric analysis, one of the most common ways is to construct structured hexahedral meshes, which have regular tensor product structure, and fit them by volumetric T-Splines. This theoretic work proposes a novel surface quadrilateral meshing method, *colorable quad-mesh*, which leads to the structured hexahedral mesh of the enclosed volume for high genus surfaces.

The work proves the equivalence relations among colorable quad-meshes, finite measured foliations and Strebel differentials on surfaces. This trinity theorem lays down the theoretic foundation for quadrilateral/hexahedral mesh generation, and leads to practical, automatic algorithms.

The work proposes the following algorithm: the user inputs a set of disjoint, simple loops on a high genus surface, and specifies a height parameter for each loop; a unique Strebel differential is computed with the combinatorial type and the heights prescribed by the user's input; the Strebel differential assigns a flat metric on the surface and decomposes the surface into cylinders; a colorable quad-mesh is generated by splitting each cylinder into two quadrilaterals, followed by subdivision; the surface cylindrical decomposition is extended inward to produce a solid cylindrical decomposition of the volume; the hexahedral meshing is generated for each volumetric cylinder and then glued together to form a globally consistent hex-mesh.

The method is rigorous, geometric, automatic and conformal to the geometry. This work focuses on the theoretic aspects of the framework, the algorithmic details and practical evaluations will be given in the future expositions.
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Keywords: Hexahedral mesh; Quadrilateral mesh; Foliation; Strebel differential

* Corresponding author at: Computer Science Department, Stony Brook University, New York, United States.
E-mail address: gu@cs.stonybrook.edu (D.X. Gu).

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1. Introduction

1.1. Motivation

Mesh generation plays a fundamental role in Computer Aided Design (CAD) and Computer Aided Engineering (CAE) fields. Finite Element Method (FEM) requires the input solids to be tessellated with high qualities. *There are mainly three types of volumetric meshing, the unstructured tetrahedral meshing, the unstructured hexahedral meshing and the structured hexahedral meshing.* Comparing to tetrahedral meshes, hexahedron mesh has many advantages [1]. The most important benefits are higher numerical accuracy, lower spacial complexity and higher efficiency:

- Non-uniform scaling hexahedra has much greater numerical accuracy compared to tetrahedra [2].
- The number of elements of a hexahedral mesh is four to ten times less than that of a tetrahedral mesh with the complexity of the input mesh being constant [2].
- Numerical computations on hexahedral meshes are up to 75% less memory and time consuming in comparison to tetrahedral meshes [3].

Automatic tetrahedral mesh generation is relatively mature, there exist reliable tools to generate high quality tetrahedral mesh automatically [4]. In contrast, automatic hexahedral mesh generation remains a great challenge, which is the so-called "holy grid" problem [2].

Recent years have witnessed the rapid development of the methodology of isogeometric analysis [5,6]. In Computer Aided Design (CAD) field, the geometric shapes are represented as Spline surfaces/solids. The most prominent Spline schemes are T-Splines [7]. In Computer Aided Engineering (CAE) field, the isoparametric philosophy represents the solution space for dependent variables in terms of the same functions which represent the geometry. In reverse engineering field [8], shapes in real life are often acquired by 3D scanning technologies and represented as point clouds. The point cloud is triangulated to generate the boundary surface, the tetrahedral mesh is generated to tessellate the interior using automatic tetrahedral meshing generation tools. In order to apply isogeometric analysis method, the solid needs to be parameterized and fitted by volumetric Splines. The hexahedral meshes for isogeometric analysis are required to have **tensor product structure** locally, and with minimal number of singular vertices or line segments.

There are different approaches for hexahedral mesh generation. One approach is to construct a quadrilateral mesh for the boundary surface, then extend the boundary mesh into the interior, and construct a hexahedral mesh for the entire solid. The main problem the current work focuses on is as follows:

Given a closed surface S, with minimal user input, automatically construct a quadrilateral mesh $\mathcal{Q}$ on S, and extend $\mathcal{Q}$ to a hexahedral mesh of the enclosed volume. Both the quadrilateral and hexahedral meshes are with local tensor product structures, and the least number of singular vertices and singular lines.

1.2. Non-structured hex-meshing

First, we consider general non-structured hex-meshing, which does not require the hex-mesh to have local tensor-product structure. The topological conditions for extending a quad-mesh to such a hex-mesh has been fully studied.

Definition 1.1 (*Extendable Quad-Mesh*)**.** Suppose Ω is a volumetric domain in $\mathbb{R}^3$, $\mathcal{Q}$ is a topological quad-mesh of its boundary surface $\partial\Omega$. If $\mathcal{Q}$ is the boundary of a topological hex-mesh of Ω, then we say $\mathcal{Q}$ is *extendable*.

One intriguing problem is to find the sufficient and necessary condition for a quadrilateral surface mesh to be extendable. Thurston [9] and Mitchel [10] proved that for a genus zero closed surface, a quadrilateral mesh is extendable if and only if it has even number of cells, furthermore Mitchel generalized the result to high genus surface cases [10]. Eppstein [11] used this existence result and proved that a linear number of hexahedra (in the number of quadrilaterals) are sufficient in such cases.

Recently, the results of Thurston, Mitchel and Eppstein have been generalized by Erickson in [12]. Erickson considers the homology of the volume (with $\mathbb{Z}_2$ coefficients), and proved the odd-cycle criterion for extendable quad-meshes:

Theorem 1.2 (*Erickson 2014 [12]*)**.** *Let Ω be a compact subset of $\mathbb{R}^3$ whose boundary $\partial\Omega$ is a (possibly disconnected) 2-manifold, and let $\mathcal{Q}$ be a topological quad mesh of $\partial\Omega$ with an even number of facets. The following conditions are equivalent:*

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