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Post-processing and visualization techniques for isogeometric analysis results

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Abstract

Isogeometric Analysis (IGA) introduced in 2005 by Hughes et al. (2005) [1] exploits one mathematical basis representation for computer aided design (CAD), geometry and analysis during the entire engineering process. In this paper we extend this concept also for visualization. The presented post-processing and visualization techniques thereby strengthen the relation between geometry, analysis and visualization. This is achieved by facilitating the same mathematical function space used for geometry and analysis also for post-processing and visualization purposes.

During non-linear analysis derivatives are incrementally computed and stored with different basis function representations. We introduce and investigate projection methods to be able to use the same function space for both displacements and stresses without loss of accuracy.

To obtain a common representation for structured and unstructured meshes like hierarchical spline, locally refined B-spline (LR B-spline) and T-spline techniques we exploit Bézier decomposition in a post-processing step resulting in a Bézier element representation and constitute it as generalized representation. The typically used unrelated (fictitious) finite element mesh representation for visualization purposes are easily replaced without changing the underlying geometry as well as the algorithmic data structure. One further benefit of the used Bézier decomposition lies in the fact that it facilitates a natural parallel implementation on Graphics Processor Units (GPUs) exploiting shader programming.

In this paper we have developed and investigated an *accurate, efficient and practical post-processing pipeline for visualization of isogeometric analysis results*. The proposed IGA visualization pipeline consists of three steps: (1) *Projection*, (2) *Bézier decomposition* and (3) *Pixel-accurate rendering*. We have tested four different projection methods. A description on how to perform Bézier decomposition of LR B-splines are given (whereas for hierarchical and T-splines this has been done before). Furthermore, the use of GPU shader programming to enable efficient and pixel-accurate visualization is detailed.

The performance of the four different projection techniques has been tested on manufactured problems as well as on realistic benchmark problems. Furthermore, the IGA visualization pipeline has been demonstrated on a number of real-world applications.

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1. Introduction

Scientific visualization understood as a method to represent visually scientific results based on multi-dimensional large scale data sets is of significant importance in various fields, especially in computational mechanics and engineering. Today, many visualization tools and instruments are available to serve special needs in each particular field, without building the connection to the underlying mathematical modeling concept for design and analysis.

The relatively new idea of isogeometric analysis [1] allows the unification of the mathematical modeling concept behind engineering design, computational analysis and simulation. The same mathematical model representation for CAD, geometry modeling and analysis generated from Non-Uniform Rational B-Splines (NURBS) can be exploited to construct a single geometric model which is used during the entire engineering process.

In order to extend the isogeometric picture, we propose to facilitate the same mathematical function space used for design and analysis also for the mathematical modeling of the visualization process (cf. Fig. 1 right). We strengthen the relation between geometry, analysis and visualization — resulting in high quality visualizations of computational results in a natural way.

Material behavior, like deformation and failure of solid structures or fluid mechanical phenomena are described through solutions of appropriate Partial Differential Equations (PDEs). In state-of-the-art methodologies Finite Element Methods (FEMs) are used in order to find approximate solutions of PDEs.

During conventional Finite Element Analysis (FEA) every processing step (CAD model design, geometry modeling with meshing and visualization) employs a different mathematical representation (cf. Fig. 1 left). CAD models are generated using a standard representation based on NURBS. For analysis a different description based on Lagrange elements is used, that approximates the geometry. In a further post-processing step, mathematically unrelated and approximately linear finite element meshes are used to visualize the computed results. As a consequence, we have to deal with the storage and data structure of different representations that makes the simulation process computational expensive, time consuming and less accurate.

In order to present visualization results of higher quality new visualization techniques have to be employed to preserve smoothness on complex and higher-order geometries. Especially, when it comes to arbitrary large scale geometric topologies represented by high-order basis functions a guaranteed accuracy is desired. For such visualizations with an arbitrary level of detail local refinement techniques like LR B-Splines [2,3], T-Splines [4,5] and hierarchical splines [6,7] can be exploited.

We address these issues by providing a platform for generalization by exploiting spline properties and techniques like Bézier decomposition to decompose structured and unstructured mesh representations into a Bézier element representation [8–10]. With respect to efficiency and accuracy, this structure can easily be incorporated into IGA visualization pipelines allowing for a parallel computing architecture and simplifying the implementation structure for local refinement technologies.

Furthermore, we focus on the challenge to embed the visualization of second-order representations like strain and stress. Both physical entities describing material behavior and failure in structural mechanics and engineering. Strains and stresses are projected during post-processing into the same spline function space in isogeometric analysis as the displacement allowing for an efficient data representation for the visualization process. As the spline basis in general is not interpolating nodal values we have to resort to other methods than simple nodal averaging like in traditional FEA. We derive methods that take full advantage of the increased regularity of the underlying spline function space.

Typically, Lagrange polynomials are the primal unknown in FEA formulations, i.e. the displacements are represented as weighted sum of the displacements in the finite element nodes:

$$u(x) = \sum_{i=1}^{n} \phi_i(x) u_i . \tag{1}$$