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Optimal and reduced quadrature rules for tensor product and hierarchically refined splines in isogeometric analysis

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Abstract

We continue the study initiated in [30] in search of optimal quadrature rules for tensor product and hierarchically refined splines in isogeometric analysis. These rules are optimal in the sense that there exists no other quadrature rule that can exactly integrate the elements of the given spline space with fewer quadrature points. We extend the algorithm presented in [30] with an improved starting guess, which combined with arbitrary precision arithmetic, results in the practical computation of quadrature rules for univariate non-uniform splines up to any precision. Explicit constructions are provided in sixteen digits of accuracy for some of the most commonly used uniform spline spaces defined by open knot vectors. We study the efficacy of the proposed rules in the context of full and reduced quadrature applied to two- and three-dimensional diffusion-reaction problems using tensor product and hierarchically refined splines, and prove a theorem rigorously establishing the stability and accuracy of the reduced rules.

Keywords: Isogeometric analysis, Optimal quadrature rules, Generalized Gaussian quadrature rules, Reduced quadrature rules
2010 MSC: 65N30, 65D30, 65D32

1. Introduction

Isogeometric analysis [17, 27] is a relatively new computational mechanics technology which aims to unify the historically separated fields of engineering analysis and computer aided design (CAD). It appears that isogeometric analysis offers several important advantages over classical finite element analysis, such as increased robustness and accuracy on a per degree-of-freedom basis [17, 27, 36], and improved spectral properties [18, 19, 29], important in hyperbolic problems. However, much work remains to be done when it comes to efficient implementation. Inspiration can be drawn from 50 years of experience with the classical finite element method. However, the structure of isogeometric stiffness and mass matrices is different and requires novel efficient numerical linear solvers, pre-conditioners, and domain decomposition methods for

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