Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 316 (2017) 1086–1156
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Superconvergent patch recovery and a posteriori error estimation technique in adaptive isogeometric analysis

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Available online 21 November 2016

Highlights

- A posteriori error estimation methodology for adaptive isogeometric analysis using LR B-splines is developed.
- Superconvergent patch recovery of gradient field on adaptive meshes is developed.
- Algorithm for computation of true superconvergent points on non-uniform adaptive meshes is provided.
- Numerical tests verify that the developed error estimator are highly efficient and asymptotically exact.

Abstract

In this article, we address adaptive methods for isogeometric analysis based on local refinement guided by recovery based a posteriori error estimates.

Isogeometric analysis was introduced a decade ago and an impressive progress has been made related to many aspects of numerical methods and advanced applications. However, related to adaptive mesh refinement guided by a posteriori error estimators, rather few attempts are pursued besides the use of classical residual based error estimators. In this article, we explore a feature common for Isogeometric analysis (IGA), namely the use of structured tensorial meshes that facilitates superconvergence behavior of the gradient in the Galerkin discretization. By utilizing the concept of structured mesh refinement using LR B-splines, our aim is to facilitate superconvergence behavior for locally refined meshes as well. Superconvergence behavior matches well with the use of recovery based a posteriori estimator in the Superconvergent Patch Recovery (SPR) procedure. However, to our knowledge so far, the SPR procedure has not been exploited in the IGA community.

We start out by addressing the existence of derivative superconvergent points in the computed finite element solution based on B-splines and LR B-splines for an elliptic model problem (1D and 2D Poisson). Then, we present some recovery procedures for improving the derivatives (or gradient) of the isogeometric finite element solution where the SPR procedure will be the main focus. In particular, we show that our SPR procedure for the improvement of derivatives fulfills the desired consistency criteria. At the end, we develop a posteriori error estimator where the improved gradient obtained from the proposed recovery procedures is used.

Numerical results are presented to illustrate the efficiency of using SPR procedure for the improvement of derivatives (or gradient) of computed solution in isogeometric analysis. Then the proposed a posteriori error estimator based adaptive refinement

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http://dx.doi.org/10.1016/j.cma.2016.11.014

methodology is tested to solve smooth and non-smooth elliptic benchmark problems. The focus is put on whether optimal convergence rates are obtained in the computed solution or not, as well as the effectivity index of the proposed error estimators.

Keywords: Isogeometric analysis; LR B-splines; NURBS; A posteriori error estimator; Adaptivity; Superconvergence

1. Introduction

Reliability and efficiency are two major challenges in simulation based engineering. These two challenges may be addressed by error estimation combined with adaptive refinements. A lot of research has been performed on error estimation and adaptive mesh refinement over the years. However, adaptive methods are not yet an industrial tool, partly because the need for a link to traditional Computer Aided Design (CAD)-systems makes this difficult in industrial practice. Here, the use of an isogeometric analysis framework introduced by Professor Thomas J.R. Hughes (The University of Texas at Austin) and co-workers [1] may facilitates more widespread adoption of this technology in industry, as adaptive mesh refinement does not require any further communication with the CAD system.

Isogeometric analysis (IGA) has been introduced in [1] as an innovative numerical methodology for the discretization of Partial Differential Equations (PDEs). The main idea was to improve the interoperability between CAD and PDE solvers. To achieve this, authors in [1] proposed to use CAD mathematical primitives, i.e., splines and NURBS, also to represent PDE unknowns. The smoothness of splines is useful in improving the accuracy per degree of freedom and solving higher order PDEs via direct approximations. Isogeometric methods have been used and tested on a variety of problems of engineering interests, see [1,2] and references therein. The development on the mathematical front started with h-approximation properties of NURBS in [3], further studies for hpk-refinements in [4] and for anisotropic approximation in [5]. The recently published article in Acta Numerica [6] is definitely an advancement in this direction.

Non-uniform rational B-splines (NURBS) are the dominant geometric representation format for CAD. The construction of NURBS are based on a tensor product structure and, as a consequence, knot insertion (which is the means for h-refinement) has a global impact on the mesh. To remedy this a local refinement can be achieved by breaking the global tensor product structure of multivariate splines and NURBS. In the current literature there are three different ways to achieve local refinement: T-splines, LR splines and hierarchical splines. In this article, we will focus on LR-splines, introduced by Dokken et al. [7]. Johannessen et al. [8] developed adaptive local refinement techniques for isogeometric finite elements based on LR B-splines. LR B-splines have been investigated and utilized together with a newly developed a posteriori error estimate by Kumar et al. [9]. Furthermore, LR B-splines have been studied in [10], extended to facilitate divergence conforming discretization for Stokes problem [11], and applied to adaptive simulation of porous media flow [12]. A comparison of LR B-splines towards hierarchical splines may be found in [13]. An algorithm for Bézier decomposition of LR B-splines may be found in Stahl et al. [14] that enables an accurate, efficient and practical post-processing pipeline for visualization of adaptive isogeometric analysis results. Readers interested in T-splines and hierarchical splines are referred to the following references: T-splines were initially introduced in [15] and their use in isogeometric analysis was first investigated in [16,17] and later a special class of analysis suitable T-spline is developed in [18]; hierarchical splines have been first introduced in [19] and studied within the isogeometric analysis in the papers [20,21] and others. Recently, there has been much progress on the topic of the generalization of splines construction which allows local refinement, but an automatic reliable and efficient adaptive refinement procedure is still one of the key issues in isogeometric analysis. To achieve a fully automatic refinement procedure to solve PDEs problem in adaptive isogeometric analysis an *a posteriori error estimate* is required. This is the subject of the current work.

1.1. A posteriori error estimations: An overview

Since 1970s several strategies have been developed to estimate the discretization error of Finite Element (FE) solutions. The first a posteriori error estimates were introduced by Babuška and Rheinboldt in 1978, see [22,23]. Since then many different error estimation techniques have been introduced. The existing techniques to obtain energy estimates may be classified into two main categories: