ARTICLE IN PRESS

Available online at www.sciencedirect.com

ScienceDirect

Comput. Methods Appl. Mech. Engrg. ■ (■■■■) ■■■–■■■

Computer methods in applied mechanics and engineering

www.elsevier.com/locate/cma

Kinetic scheme for arterial and venous blood flow, and application to partial hepatectomy modeling

Chloe Audebert[a,b,*], Petru Bucur[c,d], Mohamed Bekheit[c,e], Eric Vibert[c,e], Irene E. Vignon-Clementel[a,b,1], Jean-Frédéric Gerbeau[a,b,1]

[a] *Inria Paris, 2 rue Simone Iff, CS 42112, 75589 Paris Cedex 12, France*

[b] *Sorbonne Universités UPMC Univ Paris 6, Laboratoire Jacques-Louis Lions, 4 Place Jussieu, 75252 Paris Cedex 05, France*

[c] *Inserm Unité 1193, Villejuif, France*

[d] *CHRU, hôpitaux de Tours, Chirurgie Hépato-biliaire et Pancréatique, Transplantation Hépatique, Tours, France*

[e] *AP-HP, Hôpital Paul Brousse, Centre Hépato-Biliaire, Villejuif, France*

Highlights

- The article introduces a kinetic scheme to solve the 1D Euler equations.
- The scheme is shown to work well for both arterial and venous wall laws.
- Vessel collapse is studied.
- Liver resection of different extents is simulated with a closed-loop 1D–0D model.
- Resulting waveform changes are successfully compared with real measurements.

Abstract

The article introduces a kinetic scheme to solve the 1D Euler equations of hemodynamics, and presents comparisons of a closed-loop 1D–0D model with real measurements obtained after the hepatectomy of four pigs.

Several benchmark tests show that the kinetic scheme compares well with more standard schemes used in the literature, for both arterial and venous wall laws. In particular, it is shown that it has a good behavior when the section area of a vessel is close to zero, which is an important property for collapsible or clamped vessels. The application to liver surgery shows that a model of the global circulation, including 0D and 1D equations, is able to reproduce the change of waveforms observed after different levels of hepatectomy. This may contribute to a better understanding of the change of liver architecture induced by hepatectomy.

Keywords: Kinetic scheme; Arterial flow; Venous flow; Vessel collapse; Surgery simulation

* Corresponding author at: Inria Paris, 2 rue Simone Iff, CS 42112, 75589 Paris Cedex 12, France.

1 Authors contributed equally.

http://dx.doi.org/10.1016/j.cma.2016.07.009

0045-7825/© 2016 Elsevier B.V. All rights reserved.

1. Introduction

Liver partial ablation surgery, namely partial hepatectomy, is necessary to treat some pathologies. In order to get a functional regeneration of the liver, the weight ratio of the remaining liver to the body must be at least 0.5% for a healthy human [1]. However, the liver ablation percentage needs sometimes to be higher, in presence of large tumors for example. Post-operative liver failure may then occur due to insufficient functional liver mass.

When partial ablation is performed, the remaining liver experiences pressure and flow changes. The importance of the hemodynamics changes depends on ablation size, but their relationship remains unclear. Moreover, the remaining liver regeneration capacity seems to be impacted by the post-resection hemodynamics. A better understanding of the hemodynamics impact of hepatectomy might therefore help improve surgical practice. To contribute to this challenge, we adopt two approaches: one is based on animal experiments, the other on mathematical modeling and simulation. The present work shows that the simulations are able to reproduce, and possibly explain, some findings of the experiments.

Experiments have been performed on pigs. This species is a good animal model for our problem since its liver to body weight ratio is close to human's [2]. Pressure and flow in the main vessels of the liver have been recorded for different resection percentages. An interesting finding of these experiments was the following: at the resection time, waveform changes were observed repeatedly in the pressure and flow measured in the hepatic artery. These changes differ for 75% and 90% hepatectomy. Since it is hypothesized that there is a link between liver architecture and hemodynamics, and since liver architecture is important to understand liver regeneration, there is a strong interest in explaining these changes in pressure and flow waveforms.

A mathematical model able to reproduce this phenomenon must satisfy several requirements. First, it has to be able to capture wave propagation. A network of vessels modeled by systems of the one-dimensional (1D) hyperbolic Euler equations is a natural candidate in this respect. The liver being perfused by both arterial and venous blood, the model should be able to address both kinds of vessels. In addition, since during surgery some vessels can be clamped, the model and the numerical scheme should be able to handle the limit of vanishing cross-section area. In this work, we propose to use a kinetic scheme, in particular because of its interesting capability to preserve the positivity of the cross-section area. This scheme was originally developed for the Saint-Venant shallow water equations. To our knowledge, this is the first time that it is used to model collapsible vessels.

Second, keeping in mind that the liver receives about 25% of the cardiac output [3], hepatectomy may also influence the systemic circulation. It is therefore desirable to embed the network of 1D models within a closed-loop model of the whole circulation, including the liver. To keep a moderate complexity, this compartment can be treated with zero-dimensional (0D) models, also known as lumped-parameter models, i.e. governed by ordinary differential equations.

The paper is organized as follows. In Section 2, the hyperbolic equations are recalled and the kinetic scheme is described, along with the boundary and coupling conditions. The kinetic scheme is validated on benchmark cases, for both arterial and venous flows. In Section 3, the closed-loop 0D–1D model is presented and the effects of partial hepatectomy are studied numerically and compared with experimental observations. Section 4 ends the paper, with some conclusions and perspectives.

2. Kinetic scheme for arterial and venous blood flow

2.1. The Euler equations of hemodynamics

Blood flow in large vessels of the cardiovascular system can be represented with a collection of one-dimensional systems of nonlinear equations:

$$\begin{cases} \partial_t A + \partial_x (Au) = 0 \\ \partial_t (Au) + \partial_x (\kappa A u^2) + \dfrac{A}{\rho} \partial_x p = Ag - f(A, A_0, u). \end{cases} \tag{1}$$

The first equation corresponds to mass conservation and the second to momentum conservation. $x \in \mathbb{R}$ denotes the coordinate along the longitudinal axis of the portion of vessel, $t \in \mathbb{R}^+$ is the time, $A(x,t)$ is the vessel cross-section area, $u(x,t)$ is the mean velocity of blood through the corresponding cross-section, ρ is the fluid density assumed