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A generalized multi-resolution expansion for uncertainty propagation with application to cardiovascular modeling

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Abstract

Computational models are used in a variety of fields to improve our understanding of complex physical phenomena. Recently, the realism of model predictions has been greatly enhanced by transitioning from deterministic to stochastic frameworks, where the effects of the intrinsic variability in parameters, loads, constitutive properties, model geometry and other quantities can be more naturally included. A general stochastic system may be characterized by a large number of arbitrarily distributed and correlated random inputs, and a limited support response with sharp gradients or event discontinuities. This motivates continued research into novel adaptive algorithms for uncertainty propagation, particularly those handling high dimensional, arbitrarily distributed random inputs and non-smooth stochastic responses.

In this work, we generalize a previously proposed multi-resolution approach to uncertainty propagation to develop a method that improves computational efficiency, can handle arbitrarily distributed random inputs and non-smooth stochastic responses, and naturally facilitates adaptivity, i.e., the expansion coefficients encode information on solution refinement. Our approach relies on partitioning the stochastic space into elements that are subdivided along a single dimension, or, in other words, progressive refinements exhibiting a binary tree representation. We also show how these binary refinements are particularly effective in avoiding the exponential increase in the multi-resolution basis cardinality and significantly reduce the regression complexity for moderate to high dimensional random inputs. The performance of the approach is demonstrated through previously proposed uncertainty propagation benchmarks and stochastic multi-scale finite element simulations in cardiovascular flow.

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Keywords: Uncertainty quantification; Multi-resolution stochastic expansion; Cardiovascular simulation; Sparsity-promoting regression; Relevance vector machines; Multi-scale models for cardiovascular flow

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1. Introduction

Computational models have become indispensable tools to improve our understanding of complex physical phenomena. Recent developments of these tools enable simulation of complex multi-physics systems at a cost that is, in many cases, negligible compared to setting up a physical experiment. Recent trends have led to a transition from deterministic to stochastic simulation approaches that better account for the intrinsic variability in parameters, material constant, geometry, and other input quantities. This improved approach boosts the predictive capability of models, allowing one to statistically characterize the outputs and therefore to quantify the confidence associated with the predictions.

While promising examples of this transition can be found in a variety of application fields, in this study we focus on the hemodynamic analysis of rigid or compliant vessels in the cardiovascular system. This is a rich application field where a stochastic solution requires multiple conceptual steps, from model reduction to data assimilation, and from design of effective parameterizations (e.g., for geometry, material properties, etc.) to efficient uncertainty propagation. In this study we focus on the propagation step, where the development of a general approach raises several challenges. First, our methods must handle arbitrary random inputs, from independent inputs with non-identical distribution, to correlated samples assimilated using Markov chain Monte Carlo (MCMC) from observations of the output quantities of interest. Second, there is a need for effective adaptive algorithms that are also easy to implement. Third, it is preferable to have the ability to reuse an existing library of model solutions. And finally, there is a need for an effective approach to reconstruct a stochastic response of interest requiring a minimal number of model evaluations.

Numerous approaches for uncertainty propagation have been proposed in the literature, many of which have been designed with specific applications in mind. As a comprehensive review of these approaches is outside the scope of this paper, here we mainly focus on methodologies supporting adaptivity, efficient reconstruction of sparse stochastic responses, and use of multi-resolution representations relevant to cardiovascular simulation.

The foundations for uncertainty propagation were laid in the first half of the last century [1,2] and re-proposed in [3] in the context of intrusive stochastic finite elements. An extension of Wiener chaos expansion to non-Gaussianly distributed random inputs, was introduced in [4] associating families of orthogonal polynomials in the Askey schemes with commonly used probability measures. An analysis of the convergence properties of this scheme is also proposed in [5]. Non intrusive stochastic collocation on tensor quadrature grids was formalized in [6] for random elliptic differential equations and extended in [7,8] to isotropic and anisotropic sparse tensor quadrature rules, respectively.

Adaptivity was introduced in [9] for non-intrusive uncertainty propagation schemes using a multi-element approach, while adaptive hierarchical sparse grids were proposed in [10]. An adaptive approach based on stochastic simplex collocation was proposed in [11] with the ability of supporting random input samples characterized by non regular domains. Use of sparsity-promoting approaches to identify the polynomial chaos coefficients was proposed in [12] using standard ℓ_1 minimization, while a re-weighted ℓ_1 minimization strategy was proposed in [13]. Relevance vector machine regression in the context of adaptive uncertainty propagation was proposed in [14]. Use of multi-resolution expansion was first introduced in [15] for intrusive uncertainty propagation, and in [16] for the non-intrusive case. Finally, applications of stochastic collocation to cardiovascular simulation were proposed in [17] and in [18] in the context of robust optimization. Application to a human arterial tree with 37 parameters is also discussed in [19] using a sparse grid stochastic collocation method based on generalized polynomial chaos. Combination of data assimilation and uncertainty propagation was proposed in [20] in the context of virtual simulation of single ventricle palliation surgery.

In this paper we propose a generalized multi-resolution approach to uncertainty propagation, as an extension of the approach presented in [16]. In this multi-resolution approach, the range of the random inputs is partitioned into multiple *elements* where independent approximations of the local stochastic response are computed. These follow an expansion in an orthonormal multi-wavelet basis constructed with respect to the probability measure defined on each single element and generalize the approach in [21] which is limited to a uniform underlying measure. Expansion coefficients are computed using a Bayesian approach that has a number of advantages over previously proposed greedy heuristics for sparse regression. Based on the computed coefficients, element refinement is performed along one single dimension, leading to a significant reduction in the cardinality of multi-wavelet basis. The relevance of the proposed approach lies in its generality, ability to cope with steep gradients in the stochastic response, arbitrarily distributed random inputs and unstructured, e.g., random, solution samples.