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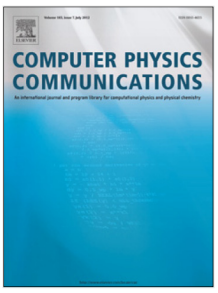

New fast accurately conservative scheme for solving numerically the time-dependent isotropic Fokker-Planck equation

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New fast accurately conservative scheme for solving numerically the time-dependent isotropic Fokker-Planck equation

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Abstract

We present a new numerical method for solving the time-dependent isotropic Fokker-Planck equation. We show analytically and numerically that the numerical scheme provides accurate particle and energy density conservation in practical conditions, an equilibrium solution close to the Maxwellian distribution, and the decrease of entropy with time. The slight nonconservation of particle and energy density is only due to the finite value of the upper bound of the energy grid. Additionally, the totally implicit scheme proves to provide positive solutions and to be unconditionally stable. The implicit forms of the scheme can be set as a nonlinear tridiagonal system of equations and solved iteratively. For a uniform grid in energy with N points, the number of operations required to compute the solution at a given time is only $O(N)$, in contrast to the totally explicit variant, which requires $O(N^3)$ operations due to the restriction on the time step. The time-centered variant is more accurate than the totally implicit one, and uses an equivalent CPU time, but does not provide positive solutions for very large timesteps. The results of the method are analyzed for the classical problem of an initially Gaussian distribution as well as for an initially quasi-truncated Maxwellian distribution.

Keywords: Fokker-Planck equation, finite differences, conservative scheme, entropy property, positivity, iterative solution, numerical stability

1. Introduction

In physics the Fokker-Planck equation (FPE) describes the time evolution of the velocity or energy distribution function of particles subjected to many small changes of velocity or energy [1, 2],