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A Drift-Asymptotic scheme for a fluid description of plasmas in strong magnetic fields

Fabrice Deluzet [a,*], Maurizio Ottaviani [b], Stefan Possanner [a]

[a] *Université de Toulouse & CNRS, UPS, Institut de Mathématiques de Toulouse UMR 5219, F-31062 Toulouse, France*
[b] *CEA Cadarache, IRFM/SCCP Bat. 513/148 13108 Saint-Paul-lez-Durance, France*

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ABSTRACT

We present a numerical scheme for the ion Euler equations with Braginskii closure, in the quasi-neutral regime with an adiabatic electron response. The scheme is constructed with the aid of asymptotic-preserving (AP) techniques in order to avoid the singularity in the drift-limit. When the normalized gyro-radius tends to zero, the scheme performs the drift-limit numerically. Depending on the choice of the time step, it can resolve different physical phenomena, ranging from cyclotron motion to plasma transport or ion drifts. Since the development of AP-schemes for the Braginskii equations is in its exploratory phase, the plasma is assumed a three-dimensional slab in a uniform external magnetic field. We use the ion-temperature-gradient dispersion relation for the scheme's verification. The promising results show that the method offers the possibility to adapt the numerical parameters to the desired resolution in the full fluid model, instead of switching to reduced models in the drift-limit.

1. Introduction

Magnetized plasmas are characterized by very disparate time and spatial scales. Consider, in particular, the toroidal confinement device known as tokamak, the leading candidate for a thermonuclear fusion reactor prototype. In its strong magnetic field of several Tesla [1], ions rotate around the field line at a cyclotron frequency ω_c of the order of hundred megahertz for the hydrogen isotopes (hundreds of gigahertz for the electrons). The typical kinetic energy of both ions and electrons for a reactor scale tokamak of several meters is in the range of 10–20 keV. At this energy, the rotation radius (Larmor radius) for the ion species is a few millimeters, which is nearly three orders of magnitude smaller than the machine scale.

Most of the interesting phenomena affecting the plasma behavior, and hence the performance of a possible future reactor, occur on timescales much longer than the cyclotron period. Magneto-hydro-dynamic (MHD) phenomena of interest develop on the microsecond–millisecond range. This is also the case for plasma turbulence, the subject addressed in this work. The overall plasma energy confinement time is of the order of a second. When modeling such plasmas, one can therefore see the interest in devising techniques that avoid treating the ion cyclotron frequency explicitly, and this has been indeed the case in the literature.

The approach is usually one that relies on analytic approximations of the fundamental equations of plasma dynamics. These equations are developed in the ratio ω/ω_c, ω being the characteristic frequency of the phenomenon one wants to study, before carrying out any numerical treatment. When starting from the fluid Braginskii equations [2], the above procedure takes the form of an expansion of the perpendicular velocity in terms of the particle drifts, and it leads to some set of *reduced fluid model equations* that is valid in the low frequency limit (see for instance [3]). When the starting point is the kinetic (Vlasov) equation, the *reduction* leads to the gyro-kinetic equation [4].

The drawback of employing reduced models is the introduction of uncertainty, due to the approximations, and the fact that the resulting equations, although free from the fast time scale, are somewhat more complicated than the original equations. The set of equations become more complex as the validity range of the reduced model is larger (see for example Yagi and Horton [5] for a broad validity model).

Reduced models were a necessity when the computational power was a small fraction of what is currently available.

Nowadays, thanks to the increased computer resources and the improvement in numerical schemes, it is becoming affordable to work directly with the more fundamental Vlasov or Braginskii models, which is desirable due to their simplicity and richer physics content. In this case, the goal to eliminate the unwanted fast time scales, still present in the more fundamental models, is achieved *numerically* (instead of *analytically*, as in the reduced model approach).

* Corresponding author.
E-mail addresses: fabrice.deluzet@math.univ-toulouse.fr (F. Deluzet), Maurizio.Ottaviani@cea.fr (M. Ottaviani), stefan.possanner@ipp.mpg.de (S. Possanner).

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In recent years, asymptotic preserving (AP) methods, of which we give a brief account below, have emerged as an effective approach to achieve such goal. In particular, in the context of plasma physics, significant case studies have been carried out (see [6,7] for a review) with, for instance, the investigation of the quasi-neutral limit of fluid plasma models [8–10] and anisotropic diffusion equations [11–14]. As an example of the effectiveness of these methods in the framework of tokamak plasma physics, the problem of the strongly anisotropic heat conduction in a rotating magnetic island [15] was successfully treated with a perpendicular to parallel conductivity ratio as low as 10^{-20}. Another example is the problem of magnetic island growth from the linear phase, through Rutherford's phase to saturation. In this instance an AP-like scheme made it possible to work with arbitrarily large Lundquist number by filtering out the Alfven frequency [16]. Still, it was shown that the very same scheme can also treat Alfven oscillations if one chooses a suitably short time step.

The goal of this work is to explore the capability of AP schemes to treat plasma turbulence. For this, we consider a specific problem, the ion temperature gradients (ITG) instability, which is at the origin of plasma turbulent convection and a major cause of plasma energy losses in a tokamak device. Given the difficulty of this task, in this first study we consider only the case of uniform magnetic field and adiabatic electrons, while still working with a Braginskii type initial model that retains the Lorentz force. While too simple to describe the situation in a fusion device, this problem presents sufficient challenges to be considered a valid test bench for the AP scheme. In particular, the model we treat contains fast time scales associated with the cyclotron motion and with the perpendicular sound waves which are not present in the corresponding reduced model, which describes only drift and quasi-parallel sound waves. We recall that it is the role of the AP scheme to filter such high frequencies out while retaining the low frequency motion of interest.

We now describe the notion of Asymptotic-Preserving methods. AP schemes have been introduced to address singular perturbation problems. This is a class of multi-scale problems with stiff terms that do not allow the computation of all of the unknowns in the limit of an infinite stiffness.[1]

As a paradigm, consider the anisotropic diffusion problem in two dimensions:

$$\partial_t \phi^\varepsilon(x,y) - \partial^2_{xx}\phi^\varepsilon(x,y) - \frac{1}{\varepsilon}\partial^2_{yy}\phi^\varepsilon(x,y) = 0, \qquad (P^\varepsilon_{sing})$$

with Dirichlet boundary conditions in x, periodic boundary conditions in y, and ε a parameter that one allows to take arbitrarily small values.

This model exemplifies the situation one finds in a magnetic confinement device, where x can be considered the radial direction, y the direction along the magnetic field, and ε the ratio of the perpendicular to the parallel diffusivity.

If one attempts to solve the above problem with, say, finite differences, and an implicit method, one finds that the condition number of the matrix of the corresponding linear problem diverges as $1/\varepsilon$. The reason is that the operator ∂^2_{yy} does not have an empty kernel (it has a zero eigenvalue since any function f independent of y satisfies $\partial^2_{yy}f = 0$.) As a consequence, the problem becomes singular when ε tends to zero. For finite ε, there is a subset of eigenvalues which do not depend on ε, whereas most of them scale like $1/\varepsilon$. Then, the ratio between the maximum and the minimum eigenvalue, which is related to the condition number, scales like $1/\varepsilon$.

This difficulty can be overcome by reformulating the problem before discretizing. In the example under consideration, this can be achieved by working with two separate functions, a 1D function representing the average of $\phi^\varepsilon(x,y)$ along y, $\bar\phi(x) = \int \phi(x,y)dy$, and a 2D function representing its deviation from the average, $\tilde\phi(x,y) = \phi(x,y) - \bar\phi(x)$.

The reformulated problem,

$$\begin{cases} \partial_t\tilde\phi^\varepsilon(x,y) - \partial^2_{xx}\tilde\phi^\varepsilon(x,y) - \dfrac{1}{\varepsilon}\partial^2_{yy}\tilde\phi^\varepsilon(x,y) = 0, \\ \partial_t\bar\phi^\varepsilon(x) - \partial^2_{xx}\bar\phi^\varepsilon(x) = 0 \end{cases} \qquad (P^\varepsilon)$$

is not singular, since the operator ∂^2_{yy} now acts on the space of zero-mean functions and has no zero eigenvalue. All the eigenvalues of the first equation scale like $1/\varepsilon$ in the limit of small ε. Then, one can estimate that the condition number now scales like N^2, where N is the number of points in the y direction. Therefore, it remains bound when ε tends to zero and the problem is treatable in this limit.

Thus, a problem reformulation is the prerequisite before numerical implementation. As above, we identify such reformulated problem as (P^ε). The corresponding limiting problem for ε tending to zero, which is now assumed well defined, is indicated by (P^0).

The goal of an Asymptotic-Preserving approach is to derive a discrete problem (P^ε_h), with h standing for the discretization parameters, which is consistent with the reformulated multi-scale problem (P^ε) when the stiffness is resolved by the numerical parameters, while offering a consistent discretization of the limit problem (P^0), for stiffness too large to be resolved.

These properties are illustrated by the diagram of Fig. 1.

$$\begin{tikzcd} P^\varepsilon_h \arrow[r, "\varepsilon\to 0"] \arrow[d, "h\to 0"] & P^0_h \arrow[d, "h\to 0"] \\ P^\varepsilon \arrow[r, "\varepsilon\to 0"] & P^0 \end{tikzcd}$$

Fig. 1. Consistency properties of Asymptotic-Preserving methods.

Seminal work was devoted to the kinetic plasma description in the limit of infinite collision rates [17–20] (see also [21,22] for a review). In this context, AP methods provide a discretization of the kinetic system or of a macroscopic (fluid) limit depending on how the reciprocal of the collision rate and the mean free path scale with respect to the time interval and the mesh space. Other singular limits have also been addressed, the low Mach asymptotics for the Euler or Navier–Stokes equations [23–25] and the drift limit of fluid plasma descriptions [26–28]. As pointed out before, the purpose of this paper is to extend the development of AP methods to the context of plasma turbulence. Compared to precedent works, this requires a model relevant for magnetized plasmas, incorporating Braginskii closures and an appropriate equation for the self consistent computation of the electrostatic field.

This article is organized as follows: in Section 2, the physical context of the plasma fluid model is precised, with the definition of the scaling assumptions leading to its dimensionless form. The asymptotic preserving reformulation is addressed in Section 3 with the numerical scheme detailed in Section 4. A stability analysis of the scheme is investigated by means of a Von Neumann analysis. Finally, Section 5 is devoted to the verification of the scheme, scanning through the characteristic plasma time scales, from the cyclotron period to the drift wave period, which are well separated in fusion plasmas [29], with a particular attention brought to the intermediate regime characterizing the ITG growth.

[1] Note that an infinite stiffness is not necessarily representative of the physics, but it permits to anticipate the property of the numerical methods operating a finite precision arithmetic.

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