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`epsilon`: A tool to find a canonical basis of master integrals ☆

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ABSTRACT

In 2013, Henn proposed a special basis for a certain class of master integrals, which are expressible in terms of iterated integrals. In this basis, the master integrals obey a differential equation, where the right hand side is proportional to ϵ in $d = 4 - 2\epsilon$ space–time dimensions. An algorithmic approach to find such a basis was found by Lee. We present the tool `epsilon`, an efficient implementation of Lee's algorithm based on the `Fermat` computer algebra system as computational back end.

Program summary
Program Title: epsilon
Program Files doi: http://dx.doi.org/10.17632/j59sy5n729.1
Licensing provisions: GPLv3
Programming language: C++
Nature of problem: For a certain class of master integrals, a canonical basis can be found in which they fulfill a differential equation with the right hand side proportional to ϵ. In such a basis the solution of the master integrals in an ϵ-expansion becomes trivial. Unfortunately, the problem of finding a canonical basis is challenging.
Solution method: Algorithm by Lee [1]
Restrictions: The normalization step of Lee's algorithm will fail if the eigenvalues of the matrix residues are not of the form $a + b\epsilon$ with $a, b \in \mathbb{Z}$. Multi-scale problems are not supported.

[1] R.N. Lee, JHEP **1504** (2015) 108 [arXiv:1411.0911 [hep-ph]].

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1. Introduction

The perturbative treatment of quantum field theories leads quite naturally to the problem of evaluating a large number of multi-loop Feynman diagrams. After a tensor reduction the Feynman diagrams can be expressed in an even larger number of scalar Feynman integrals of the form

$$\int d^d l_1 \dots \int d^d l_L \frac{1}{D_1^{n_1} \dots D_N^{n_N}}, \tag{1}$$

where L is the number of loops and $d = 4 - 2\epsilon$ the number of space–time dimensions in the context of dimensional regularization. The denominators D_i in (1) are usually of the form $p^2 - m^2$, where p is a linear combination of loop momenta and external momenta and m some mass.

A standard technique nowadays is the usage of integration-by-parts identities [1,2] for the reduction of this large number of Feynman integrals to a rather small set of so-called master integrals. These identities provide linear dependences between various Feynman integrals, where the coefficients are rational functions in both the space–time dimension d and the kinematic variables of the problem.

Many methods were developed to solve these master integrals. For an overview see e.g. [3]. Among the most successful ones is the method of differential equations which is also based on integration-by-parts reductions [4–6]. Recently, significant progress was made in this method, when Henn conjectured the existence of a canonical basis for master integrals expressible in terms of iterated integrals [7]. In this basis the right hand side of the system of differential equations is proportional to $\epsilon = (4 - d)/2$. If the boundary conditions are known, the solution of the system of differential equations in an ϵ-series becomes trivial.

☆ This paper and its associated computer program are available via the Computer Physics Communication homepage on ScienceDirect (http://www.sciencedirect.com/science/journal/00104655).

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Two years ago, Lee proposed an algorithm to automate finding a canonical basis [8]. A first implementation for this algorithm was presented in [9].

In this paper we present `epsilon`, a further implementation of Lee's algorithm based on the `Fermat`[10] computer algebra system. Our implementation utilizes the explicit dependence of the transformations used by Lee's algorithm on the kinematic variable to reduce the number of variables in intermediate steps. Another advantage of our implementation is the support of systems with singularities at complex points using `Fermat`'s polymod capability.

In Section 2 we introduce some definitions and explain implementation details. In Section 3 the installation procedure and the usage of `epsilon` is described. In Section 4 we give a non-trivial example of the usage based on a real three-loop computation.

2. Implementation details

2.1. Definitions

We consider a set of N master integrals $\vec{f}$ fulfilling an ordinary system of differential equations

$$\frac{\partial \vec{f}(x,\epsilon)}{\partial x} = \mathbb{M}(x,\epsilon)\vec{f}(x,\epsilon), \tag{2}$$

where x is a kinematic variable, $\mathbb{M}(x,\epsilon)$ is an $N \times N$-matrix and ϵ is a regulator in $d = 4 - 2\epsilon$ dimensions in the context of dimensional regularization. We restrict ourselves to the case

$$\mathbb{M}(x,\epsilon) = \sum_{x_j \in S}\sum_{k\geq 0} \frac{\mathbb{M}_k^{(x_j)}(\epsilon)}{(x-x_j)^{k+1}} + \sum_{k\geq 0} x^k \mathbb{M}_k(\epsilon), \tag{3}$$

where S is the set of all finite singularities and $\mathbb{M}_k^{(x_j)}$ and $\mathbb{M}_k(\epsilon)$ are independent of x. In particular, singularities x_j depending on ϵ are forbidden. In many physically relevant cases one can use a trial and error approach to find a basis of master integrals $\vec{f}$ fulfilling the restriction (3). The main strategy of our implementation is to keep the system always in the form of (3) since here the x-dependence is explicit.

A singularity $x_j < \infty$ has Poincaré rank p if $\mathbb{M}_p^{(x_j)} \neq 0$ and $\mathbb{M}_k^{(x_j)} = 0$ for $k > p$. In addition to the finite singularities, the system might also have a singularity at ∞. The Poincaré rank p of a singularity at ∞ is defined as the Poincaré rank of the singularity at $y = 0$ of the system $\mathbb{M}(1/y,\epsilon)/y^2$. So (3) has Poincaré rank $p > 0$ at ∞ if $\mathbb{M}_{p-1} \neq 0$ and $\mathbb{M}_k = 0$ for $k \geq p$, and Poincaré rank $p = 0$ at ∞ if all $\mathbb{M}_k = 0$ and $\sum_{x_j\in S}\mathbb{M}_0^{(x_j)} \neq 0$. If all $\mathbb{M}_k = 0$ and $\sum_{x_j\in S}\mathbb{M}_0^{(x_j)} = 0$, the system is not singular at ∞.

Let p be the Poincaré rank of a singularity $x_j < \infty$, then the generalized Poincaré rank (or Moser rank) [11] of this singularity is defined as $p + r/n - 1$, where $r = \operatorname{rank} \mathbb{M}_p^{(x_j)}$ and n is the dimension of the system.

A system

$$\mathbb{M}(x,\epsilon) = \sum_{x_j\in S} \frac{\mathbb{M}_0^{(x_j)}(\epsilon)}{x - x_j}, \tag{4}$$

where all singularities have Poincaré rank zero is called Fuchsian, and a system

$$\mathbb{M}(x,\epsilon) = \epsilon \sum_{x_j\in S} \frac{\widehat{\mathbb{M}}_0^{(x_j)}}{x - x_j}, \tag{5}$$

where $\widehat{\mathbb{M}}_0^{(x_j)}$ is no longer a function of ϵ, is said to be in ϵ-form. A change of basis

$$\vec{g}(x,\epsilon) = \mathbb{T}^{-1}(x,\epsilon)\vec{f}$$

modifies the system (2) to

$$\frac{\partial \vec{g}(x,\epsilon)}{\partial x} = \widetilde{\mathbb{M}}(x,\epsilon)\vec{g}(x,\epsilon),$$

with

$$\widetilde{\mathbb{M}}(x,\epsilon) = \mathbb{T}^{-1}(x,\epsilon)\mathbb{M}(x,\epsilon)\mathbb{T}(x,\epsilon) - \mathbb{T}^{-1}(x,\epsilon)\frac{\partial}{\partial x}\mathbb{T}(x,\epsilon). \tag{6}$$

We assume the master integrals in $\vec{f}$ to be ordered in a way that a block-triangular structure of the system is obtained (for details see e.g. [8]). We will often make use of this block-triangular structure. Therefore we write

$$\mathbb{M} = \begin{pmatrix} \mathbb{A} & 0 & 0 \\ \mathbb{B} & \mathbb{C} & 0 \\ \mathbb{D} & \mathbb{E} & \mathbb{F} \end{pmatrix}, \tag{7}$$

and use the same indices as in (3) for the matrices $\mathbb{A}, \ldots, \mathbb{F}$ (e.g. $\mathbb{C}_k^{(x_j)}(\epsilon)$). The block $\mathbb{C}$ is called the active block as we apply Lee's algorithm to this block. As $\mathbb{A}$ to $\mathbb{F}$ are matrices as well, the definition of what we call the active block is more or less arbitrary as long as a block-triangular structure is obtained. But from a computational point of view a small dimension of the active block is preferable since this reduces the complexity of the resulting operations. In the following, the matrices $\mathbb{A}_k^{(x_j)}, \ldots, \mathbb{F}_k^{(x_j)}$ and $\mathbb{A}_k, \ldots, \mathbb{F}_k$ will be referred to as coefficient matrices.

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