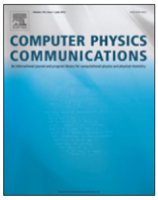
Computer Physics Communications 210 (2017) 10–28

Contents lists available at ScienceDirect

Computer Physics Communications

journal homepage: www.elsevier.com/locate/cpc



A truly incompressible smoothed particle hydrodynamics based on artificial compressibility method

Fardin Rouzbahani [a], Kazem Hejranfar [b,*]

[a] Department of Mechanical and Aerospace Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran
[b] Department of Aerospace Engineering, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 20 March 2015
Received in revised form
16 August 2016
Accepted 15 September 2016
Available online 29 September 2016

Keywords:
Incompressible flows
Smoothed particle hydrodynamics
Chorin's artificial compressibility method

ABSTRACT

In the present study, a truly incompressible smoothed particle hydrodynamics based on the artificial compressibility method for simulating steady and unsteady incompressible flows is proposed and assessed. The incompressible Navier–Stokes equations in the primitive variables formulation using the artificial compressibility method proposed by Chorin in the Eulerian reference frame are written in a Lagrangian reference frame to provide an appropriate incompressible SPH algorithm. The proposed SPH formulation implemented here is based on an implicit dual-time stepping scheme to be capable of time-accurate analysis of unsteady flows. The advantage of the Artificial Compressibility-based Incompressible SPH (ACISPH) method proposed over the Weakly Compressible SPH (WCSPH) and the pseudo-compressibility SPH methods is that the ACISPH formulation is a truly incompressible SPH algorithm and it does not involve any approximate enforcement of the incompressibility condition that usually causes to use a large magnitude speed of sound implying a small time step in the computations. The approximate enforcement of the incompressibility condition used in the WCSPH method also causes spurious oscillations in the density and pressure fields which is not the case for the ACISPH formulation proposed. Unlike the projection SPH algorithm and the moving-particle semi-implicit (MPS) method formulation, the ACISPH formulation presented herein does not involve the iterative solution of a Poisson equation for the pressure field. The accuracy and robustness of the proposed incompressible SPH (ACISPH) are demonstrated by solving different incompressible flow problems. A sensitivity study is also conducted to evaluate the effects of particle resolution and the value of artificial compressibility parameter on the accuracy and convergence rate of the solution. To validate the results, different test cases are considered herein that are incompressible flows in a 2-D simple channel, a 2-D locally expanded channel, a 2-D cavity, the unsteady Couette flow with pressure gradient, the Taylor vortex problem and a dam break problem with dry bed. Results obtained for these cases are in good agreement with the available analytical and numerical results. The study shows the artificial compressibility-based ISPH (ACISPH) method proposed is accurate and robust for simulating steady and unsteady incompressible flow problems.

1. Introduction

In computational fluid dynamics (CFD), many grid-based numerical methods are developed for solving and studying different fluid flow problems. The main effort in this issue is usually solving the Navier–Stokes equations in a common numerical way. Although these methods have shown their ability in simulating many problems, the existence of some fairly complicated problems with free surfaces, large deforming boundaries, etc., makes modeling of them very difficult when using grid-based numerical methods. Note that the generation of a suitable mesh in grid-based methods for complex geometries is not an easy task and it usually involves a complicated and time-consuming procedure. In addition, the precise calculation of free surfaces and deforming boundaries cannot be easily performed by applying these methods. Thus, efforts are continuing to develop and employ other ways to overcome these difficulties. Smoothed particle hydrodynamics (SPH) method is one of these ways that is a meshfree method. SPH for the first time was used in astrophysical problems by Lucy [1], and Gingold and Monaghan [2]. The SPH method has been successfully applied to numerous scientific applications. Like most other numerical schemes, however, SPH experiences continual theoretical and technical developments.

---

* Corresponding author. Fax: +98 2166022731.
E-mail address: khejran@sharif.edu (K. Hejranfar).

http://dx.doi.org/10.1016/j.cpc.2016.09.008

Numerical simulation of incompressible fluid flow, due to its many industrial applications, has a great practical significance. Monaghan [3] used the SPH method to simulate incompressible free surface flows. In the past two decades, considerable efforts have been made to simulate various problems using SPH, for example, fluid dynamics [4,5], and heat transfer problems [6,7], viscoelastic flows [8,9], solid mechanics [10,11] and multi-phase flows [12,13]. A comprehensive review of these methods and recent developments can be found in [14,15]. Recently, some SPH solvers have been developed due to their capability of modeling of complex flows (for example, see Ref. [16] and the other SPH solvers introduced there).

Two different approaches are usually adopted in SPH for incompressible fluid flows modeling:

(1) **The weakly compressible SPH (WCSPH) method**

This method was presented at first by Monaghan [3] and he used a compressible flow solver. To satisfy the incompressibility condition, the speed of sound is chosen to be large enough; this amount usually is 10 times the maximum velocity calculated in the flow field which leads to a Mach number smaller than 0.1 and ensures that relative changes of density be lower than 1%. This approach has been widely used to solve the multi-phase flows [6,12], incompressible viscous flows [5] and non-viscous free surface flows [17]. Although the WCSPH is a simple and practical approach, the use of a large magnitude speed of sound implies a very strict CFL condition on the time step in the calculations and approximate enforcement of the incompressibility condition causes spurious oscillations in the density and pressure fields [18–20]. A comparison between the WCSPH and incompressible SPH methods can be found in Refs. [20–22]. Some efforts have been made in the literature to improve the accuracy and robustness of the WCSPH method (for example see Refs. [22–25]).

Some works have considered the application of quasi- or pseudo-compressibility in SPH framework to model the incompressible flow as a slightly compressible flow (see Refs. [5,26]). The compressible form of the governing equations is used in this SPH formulation and a quasi-incompressible (or artificial) equation of state (EOS) is applied to couple the pressure and density variables. The value of speed of sound in this formulation should be selected carefully to obtain an accurate and efficient solution. In [5], it was mentioned that the computed pressures are comparable with other methods when the speed of sound is selected such that the density varies by at most 3%. The main difference between the WCSPH and pseudo-compressible SPH methods is in the equation of state (EOS) type used in these SPH methods. The first one uses a thermodynamic EOS, while the second one employs an artificial EOS. Note that, similar to the WCSPH method, the quasi-incompressible SPH formulation cannot be considered as a truly incompressible SPH method because the density appears in this SPH formulation and its value should be calculated in the entire flowfield through solving the governing equations.

(2) **The SPH projection (PSPH) method**

This approach was proposed at first by Cummins and Rudman [18]. In this approach, similar to other forms of pressure-based methods, the pressure field is obtained by solving a Poisson equation in which the source term is proportional to the velocity divergence. The resulting flowfield satisfies the divergence-free velocity condition, however, a constant density condition is not automatically fulfilled [18] in this SPH formulation. Shao and Lo [23] modified the above method so that the changes in density are considered at the source term to apply the constant density condition and they introduced this approach as an incompressible SPH (ISPH) method. An attempt for satisfying both the divergence-free velocity and constant density conditions through a double correction (two Poisson equations) has been performed by Pozorski and Wawrenczuk [27].

These SPH projection methods for calculating the pressure require the solution of a system of linear equations at each time step. Note also that in these methods, the necessity of a sound speed is removed and thus the size of time step is larger than in the WCSPH method. Lee et al. [20] compared these two approaches on various incompressible flow test cases and concluded that the WCSPH method exhibits strong spurious oscillations especially in pressure and produces unreliable results particularly on coarse particle resolutions. Moreover, the CPU time with the incompressible SPH method was anywhere between 2 and 20 times greater than that of the WCSPH approach.

The moving-particle semi-implicit (MPS) method

There are some other Lagrangian methods developed in the literature for simulating incompressible fluids, e.g. the moving-particle semi-implicit (MPS) method (for example, see [28–30]). In MPS, the governing equations are discretized based on particle interaction models representing gradient, divergence and Laplacian by using a kernel function to interpolate the unknowns. All particle interaction models in the MPS method are macroscopic and deterministic. Computational grids are not necessary in MPS and thus it is a meshfree method similar to SPH.

The main difference between SPH and MPS is that SPH is based on an integral description of flow variables and spatial derivatives in the governing equations and it utilizes the derivative of a kernel function attached to each particle for the spatial derivatives to be evaluated, while the MPS method is based on the Taylor series expansion and the kernel function is arranged for evaluating the spatial derivatives of the governing equations in a Lagrangian frame by particle interaction models.

In the original MPS method, the incompressibility is satisfied by solving the pressure Poisson equation by applying an iterative method in each time step and it usually requires a considerable amount of the computation time [28–30]. The weakly compressible MPS (WC-MPS) [30] has been developed to alleviate this problem, however it causes the same disadvantages of the WCSPH method mentioned before. Note also that in the MPS method, it is necessary to generate the neighbor list for recording the neighboring particle numbers and the distances of all pairs of particles should be also calculated to provide the list. Such neighboring search algorithm also takes a considerable computation time in each time step, especially when requiring a large number of particles [28–30].

The artificial compressibility-based ISPH (ACISPH) method proposed

In this study, a truly incompressible SPH based on the artificial compressibility method (ACISPH) is proposed and assessed. The artificial compressibility method, introduced at first by Chorin [31] for solving the Navier–Stokes equations in the Eulerian (grid-based) reference frame, provides a mechanism to advance the flowfield in artificial time for satisfying the divergence-free velocity condition in a manner that the mass and momentum equations are conserved when a steady-state condition in artificial time is obtained. Note that Chorin's artificial compressibility approach changes the nature of the governing equations from the mixed elliptic/parabolic into a system of hyperbolic or parabolic equations in artificial time that can be solved by efficient time-marching schemes. Herein, the incompressible Navier–Stokes equations based on Chorin's artificial compressibility method are reformulated in a Lagrangian reference frame to perform an appropriate incompressible SPH algorithm. As stated before, the artificial compressibility approach has been applied in the framework of SPH in the literature but as the equation of state (EOS) for coupling the pressure and density variables in the compressible SPH formulation. In fact, the quasi-incompressible SPH and WCSPH formulations known in the literature use the compressible form of Navier–Stokes equations and they do not