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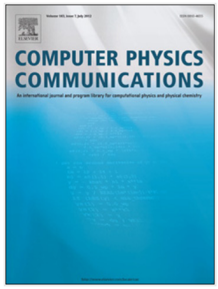

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L. Gr. Ixaru

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Numerical approach of some three-body problems

L. Gr. Ixaru

"Horia Hulubei" National Institute of Physics and Nuclear Engineering, Department of Theoretical Physics, P.O. Box MG-6, Bucharest, Romania
Academy of Romanian Scientists, 54 Splaiul Independenţei, 050094, Bucharest, Romania

Abstract

On taking the two-proton decay in nuclear physics as a sample case we formulate a numerical method for three-body problems which are mathematically described by systems of coupled 2d Schrödinger equations in polar coordinates r, ϕ. With some minimal adaptations the method becomes applicable on other cases. The specific feature of the procedure consists in a shuttle propagation along r: two quantities are propagated, the log-derivative matrix and the solution itself, with a backwards propagation of the former, followed by a forwards propagation of the latter. The results show remarkable stability. Numerical illustrations from a simple test model are reported and we also explain how the data obtained in this way can be exploited to obtain useful physical information.

Keywords: system of coupled 2d Schrödinger equations, polar coordinates, shuttle propagation, CP methods

1. Introduction

In the earlier sixties Goldansky [1] advanced the idea that a new kind of radioactivity is possible from some nuclei. This is the emission of a pair of correlated protons, two-proton decay, for short. In the following decades this exotic decay mode received much attention from experimentalists and theorists, with the hope that much valuable information could be obtained in this way, see, e.g., [2, 3, 4, 5, 6, 7] and references therein.

Seen from the perspective of a theorist this is a three-body problem (residual nucleus plus the two protons) and a popular approach in this area is that of Grigorenko et al. [4, 5, 6]. Thus, as described in [4], a Jacobi system of coordinates is used for start, and this is next converted to a hyperspherical system. The wave function is then developed on the orthonormal set of hyperspherical functions, with coefficients which depend on the hyperradius. These coefficients are found to satisfy a system of coupled 1d Schrödinger equations, and this has to be solved numerically.

Another approach is that of Delion et al. [7] where familiar spherical coordinates are used for each proton and the wave function is developed over the set of two-proton azimuthal harmonics to obtain a system of coupled 2d Schrödinger equations in the radial coordinates r_1 and r_2. These authors advocate that the approach becomes more transparent if the radial coordinates are replaced by polar coordinates r, ϕ. Indeed, the domain now becomes a rectangle and the asymptotic boundary conditions can be readily formulated. The system to be solved in the new coordinates remains a system of coupled 2d Schrödinger equations, of course.

To approach the latter problem numerically the authors of [7] use an approximate procedure for the treatment in variable ϕ. A set of values $\phi_1, \phi_2 \ldots$, is fixed and the equation in r, which is actually a system of coupled 1d Schrödinger equations, is solved for each ϕ_i. The computation is fast and the results are encouraging but the problem of how such results compare with those from a correct approach in both variables remains open.

An algorithm for systems of coupled 2d equations in polar coordinates is then needed, and this is what we do in this paper. In doing so we hope that such a method, with some minor adaptations, if any, will become of use in many other three-body problems of direct interest. As a matter of fact, the present version needs no modification for the two-neutron decay.

The method has the versions for a single 2d equation in cartesian coordinates [8] and then extended in [9] for polar coordinates as its precursors, but what is central in the new method is the insistence on its shuttle character. Two quantities are propagated along r, the log-derivative matrix of the solution and the solution itself,

Email address: `ixaru@theory.nipne.ro` (L. Gr. Ixaru)