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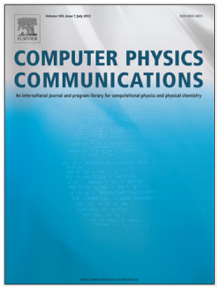

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A new data assimilation technique based on ensemble Kalman filter and Brownian bridges: an application to Richards' equation

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Abstract

In this paper a new data assimilation technique is proposed which is based on the ensemble Kalman filter (EnKF). Such a technique will be effective if few observations of a dynamical system are available and a large model error occurs. The idea is to acquire a fine grid of synthetic observations in two steps: 1) first we interpolate the real observations with suitable polynomial curves; 2) then we estimate the relative measurement errors by means of Brownian bridges.
This technique has been tested on the Richards' equation, which governs the water flow in unsaturated soils, where a large model error has been introduced by solving the Richards' equation by means of an explicit numerical scheme. The application of this technique to some synthetic experiments has shown improvements with respect to the classical ensemble Kalman filter, in particular for problems with a large model error.

Keywords: data assimilation, ensemble Kalman filter, Brownian bridges, Richards' equation.

1. Introduction

Data assimilation is a growing research field which aims to incorporate observations into a dynamical model in order to improve the modelling of an evolutionary process, usually described by a differential system. Typically, data assimilation serves two different goals: to better estimate the states of the system or to retrieve model parameters of the system itself (see, for example, [9, 2, 28]).

Here, the data assimilation procedure is used just to correct the system states between one observation and the following one. This means that we include the uncertainty on the parameters in the general expression of *model error*, whose heuristic definition is provided in the following; this is a reasonable situation, for example, in the context of groundwater modeling (see Section 4). The data assimilation technique we use stems from the ensemble Kalman filter (denoted by EnKF) that will be briefly resumed in Section 2.

Any mathematical representation of a complex problem, as the infiltration process in the unsaturated zone, suffers from a hardly quantifiable error, arising also from the reduction of the problem into a simplest model or from neglecting some aspects or approximating some others.
The approach suggested hereby is well suited when a good confidence is placed in the observations and, viceversa, a large model error occurs, whatever its cause is.
Citing [14], the general expression of *model error* takes into account any discrepancy between the simulated environment and the true environment, or, according to [17], it can be defined as "the mismatch between the model used to filter and the source of the data itself".
As a matter of fact, the model error can have different origins (see ad example [11]): numerical instabilities, chaotic nature of the model, inaccurate choice of model parameters. In this context, the data assimilation

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