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Development of the fluid-type transport code on the flux coordinates in a tokamak

Mitsuru Honda [a,*], Atsushi Fukuyama [b]

[a] National Institutes for Quantum and Radiological Science and Technology, Naka, Ibaraki 311-0193, Japan

[b] Graduate School of Engineering, Kyoto University, Kyoto 615-8246, Japan

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ABSTRACT

The one-dimensional fluid-type transport code, TASK/TX, is developed compatible with the flux coordinates in a tokamak. Unlike diffusive transport equations usually adopted in conventional transport codes, the governing equations conform to a two-fluid model consisting of Maxwell's equations and the multiple fluid moment equations for each species.Quasi-neutrality and ambipolar flux conditions are not imposed, which are inherently satisfied as a consequence of the equation system solved. The neoclassical particle flux is not approximated by the flux–gradient relationship, and the total particle flux composed of the neoclassical and turbulent contributions is directly treated as the dependent variable. The quantities related to neoclassical transport are intrinsically calculated without external neoclassical transport modules. In other words, TASK/TX by itself has the function of a neoclassical transport solver based on the moment approach as well. Several numerical tests clearly reveal the unique features of TASK/TX not possessed by conventional transport codes.

1. Introduction

Analyzing plasmas in current experiments and developing operation scenarios in future devices over the entire time of discharges, we believe that performing a transport simulation using a one-dimensional transport code is still virtually the sole solution. In an early stage of research, only heat transport together with the current diffusion was focused on using a transport code. After that, the research area has been expanded to particle transport and, in recent years, toroidal momentum transport. Each transport process is more or less approximated so that it can be treated by a transport code, and is essentially described by a convection–diffusion equation based on the classical [1] and neoclassical transport theory [2,3]. As clearly written in [2], each transport equation originates from some velocity moments of the kinetic equation: taking the velocity moments leads to the moment equations. Combining them with Maxwell's equations realizes a closed set of equations that can describe the temporal and spatial evolution of a plasma. Such a system is sometimes called a two-fluid system, which is complete and self-consistent, but is too nonlinear to be solved. Therefore, these equations are reduced to derive the diffusive transport equations that are composed of the governing equations of conventional transport codes, with several assumptions explicitly introduced: Fick's law, quasi-neutrality, ambipolarity of the radial particle flux and so forth.

To advance the transport modeling beyond the diffusive model, we have been developing a one-dimensional fluid-type transport code TASK/TX [4]. The code is essentially based on a two-fluid model, which consists of the conservation laws of particle, momentum and energy plus Maxwell's equations. It also includes the equations for beam ions [5,6] and neutrals [7]. It differs from conventional diffusive transport codes mainly in that: the quasi-neutrality condition $n_\mathrm{e} = \sum_s Z_s n_s$ and the ambipolar flux condition $\sum_s \Gamma_s^\rho = 0$ need not be imposed, where n_s is the density for species s, Z_s is the charge number, ρ is the normalized minor radius and Γ_s^ρ is the contravariant component of the particle flux; Γ_s^ρ, treated as the dependent variable, mainly consists of the neoclassical and turbulent particle fluxes and the neoclassical part is not approximated by the flux–gradient relationship; the neoclassical transport characteristics are internally reproduced without the need of an external neoclassical transport solver. There are several achievements obtained by TASK/TX which may not be obtained by conventional transport codes. The spin-up of toroidal rotation in the direction counter to the plasma current in a tokamak with the toroidal magnetic field ripple has been qualitatively reproduced by simply imposing losses of fast ions due to ripple [5]. The point

* Corresponding author.
E-mail address: honda.mitsuru@qst.go.jp (M. Honda).

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is that any torque sources regarding this mechanism were not directly given in simulations, but just the fast-ion loss channel was added to the equations of fast ions. The non-ambipolar loss of fast ions produces the fast-ion radial current, which is then compensated by the return radial current in a thermal plasma, inducing the $\boldsymbol{j} \times \boldsymbol{B}$ torque that imparts counter rotation. In a similar manner, it has been confirmed that applying the slightly different fast ion and accompanying electron source profiles, which stem from ionization of beam neutrals, due to the difference in the drift orbit produces the $\boldsymbol{j} \times \boldsymbol{B}$ torque [6]. The characteristics of the evolution of the radial electric field E_r were also investigated [8]. A main drawback of the code is, however, that the governing equations are built on the cylindrical coordinates (r, θ, ϕ), which is equivalent to the large aspect ratio limit of a plasma. In this sense, some physics originating from toroidicity such as the Pfirsch–Schlüter contribution to the flux has been dropped. Furthermore, due to the formulation on the cylindrical coordinates, compatibility of the governing equations with the existing theory established on the flux coordinates in tokamaks is low. Remodeling the governing equations of the former TASK/TX built on the cylindrical coordinates is indispensable for applying TASK/TX to physics research with actual tokamak equilibria and to analyzes and predictions of the plasma behavior and evolution.

As a model that describes transport processes, a two-fluid model is more advanced than a diffusive transport model in that it involves more physics that is reproducible. However, before focusing on the issues which a diffusive transport model cannot deal with, we should first focus on establishing governing equations on the flux coordinates based on a two-fluid model so as to reproduce all phenomena which existing diffusive transport equations can cover. In parallel with this procedure, compatibility with existing transport theory has to be carefully examined. To that end, we believe that it is important to minutely derive a new set of governing equations, sometimes revisiting the derivation of the seemingly prevalent equations and relationship to evidently illustrate what ordering and assumption are adopted. Even though the method of numerical implementation has already been described in detail in [4], it will be again described in this paper with an emphasis on the updated points. In the following derivation and coding, we assume a pure plasma consisting of electrons and single-species bulk ions, typically deuterium, although this assumption will be relaxed in future work.

This paper is organized as follows. Section 2 specifies the governing equations solved in TASK/TX in detail. In Section 3 numerical schemes adopted are briefly described. Section 4 is devoted to numerical results which stand out the characteristics of TASK/TX, different from other conventional transport codes. Finally, conclusions and discussion are given in Section 5.

2. Derive the governing equations

2.1. Maxwell's equations

In the straight field-line coordinates, the magnetic field $\boldsymbol{B}$ can be written as

$$\begin{aligned} \boldsymbol{B} &= \nabla\psi \times \nabla(q\theta - \zeta) = \nabla \times (\psi_t \nabla\theta - \psi\nabla\zeta) \\ &= \nabla\zeta \times \nabla\psi + \nabla\psi_t \times \nabla\theta, \end{aligned} \tag{1}$$

where ψ and ψ_t are the poloidal and toroidal magnetic flux functions and θ and ζ are the poloidal and toroidal angles, respectively. The safety factor q is defined as

$$q \equiv \frac{d\Psi_t}{d\Psi_p} = \frac{d\zeta}{d\theta} = \frac{\boldsymbol{B}\cdot\nabla\zeta}{\boldsymbol{B}\cdot\nabla\theta} = \frac{B^\zeta}{B^\theta}, \tag{2}$$

where Ψ_p and Ψ_t are the poloidal and toroidal magnetic fluxes defined by $\Psi_p \equiv 2\pi\psi$ and $\Psi_t \equiv 2\pi\psi_t$, respectively. Comparing Eq. (1) to $\boldsymbol{B} = \nabla\times\boldsymbol{A}$, we have the vector potential in the form: $\boldsymbol{A} = \psi_t\nabla\theta - \psi\nabla\zeta$ and thus $A_\rho = 0$, $A_\theta = \psi_t$ and $A_\zeta = -\psi$. Here, the subscripts (superscripts) of the coordinates denote the covariant (contravariant) components of a vector, and the flux surface label ρ is defined as

$$\rho \equiv \sqrt{\frac{\Psi_t}{\Psi_{ta}}}, \tag{3}$$

where Ψ_{ta} is Ψ_t at the plasma surface. It is found in the relationship that the covariant components of a vector potential are equivalent to the magnetic flux functions and subsequently are the flux functions. In axisymmetric devices like tokamaks, the toroidal coordinate ζ is taken to be equivalent to the geometrical toroidal angle and then the partial orthogonality, i.e., $\nabla\rho\cdot\nabla\zeta = 0 = \nabla\theta\cdot\nabla\zeta$, is satisfied. $\boldsymbol{B}$ is thus expressed as

$$\boldsymbol{B} = \boldsymbol{B}_p + \boldsymbol{B}_t = \nabla\zeta \times \nabla\psi + I\nabla\zeta. \tag{4}$$

Here, $I(\rho) \equiv RB_t$ and this is the definition of the toroidal magnetic field strength B_t. Note that the poloidal current function I is the flux function. From the definition, the toroidal magnetic flux can be written as

$$\Psi_t \equiv \iint_{S_\zeta} \boldsymbol{B}\cdot d\boldsymbol{S}_\zeta = \frac{1}{2\pi}\iiint_V \boldsymbol{B}\cdot\nabla\zeta\, dV = \frac{1}{2\pi}\iiint_V \frac{I}{R^2}\, dV, \tag{5}$$

where the final equality follows from Eq. (4). Here, R is the major radius and V is the plasma volume. Averaging this equation over the flux surface after differentiating it with respect to V yields

$$\frac{d\Psi_t}{dV} = \frac{1}{2\pi}\frac{d}{dV}\iiint_V \frac{I}{R^2}\, dV = \frac{I\langle R^{-2}\rangle}{2\pi}, \tag{6}$$

where we have applied the definition of the flux surface average represented by $\langle\ \rangle$ in the final equality. This equation leads to the expressions of the following flux functions:

$$I = \frac{4\pi^2}{V'\langle R^{-2}\rangle}\frac{\partial\psi_t}{\partial\rho}, \tag{7}$$

$$q = \frac{d\psi_t}{d\psi} = \frac{I\langle R^{-2}\rangle}{4\pi^2}\frac{\partial V}{\partial\psi} = \frac{IV'\langle R^{-2}\rangle}{4\pi^2}\frac{\partial\rho}{\partial\psi}, \tag{8}$$

where the differential volume V' is defined as $V' \equiv \partial V/\partial\rho$. Once ψ and ψ_t are determined by solving Maxwell's equations, I and q are determined as well.

From Faraday's law, the electric field is given by

$$\boldsymbol{E} \equiv -\nabla\Phi - \frac{\partial\boldsymbol{A}}{\partial t}, \tag{9}$$

where Φ is the electrostatic potential. The contravariant radial component of $\boldsymbol{E}$ is

$$E^\rho \equiv \boldsymbol{E}\cdot\nabla\rho = -\nabla\Phi\cdot\nabla\rho = -|\nabla\rho|^2\frac{\partial\Phi}{\partial\rho}, \tag{10}$$

where Φ has been assumed to be the flux function. As seen from the above relationship, E^ρ is not the flux function, albeit Φ is. Therefore, there are several ways to give the definition of E_r. In this paper, we use the definition of E_r in the form:

$$E_r \equiv \langle\boldsymbol{E}\cdot\nabla r\rangle = -\frac{\partial r}{\partial\rho}\langle|\nabla\rho|^2\rangle\frac{\partial\Phi}{\partial\rho}. \tag{11}$$

Here, r is the minor radius and is defined using the volume as $r \equiv \sqrt{V/(2\pi^2R)}$. Taking the inner product of $\boldsymbol{E}$ with $\nabla\zeta$ gives the contravariant component of the toroidal electric field

$$E^\zeta = -\frac{\partial A^\zeta}{\partial t} = \frac{1}{R^2}\frac{\partial\psi}{\partial t}. \tag{12}$$