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Most suitable mother wavelet for the analysis of fractal properties of stride interval time series via the average wavelet coefficient method

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ABSTRACT

Human gait is a complex interaction of many nonlinear systems and stride intervals exhibiting self-similarity over long time scales that can be modeled as a fractal process. The scaling exponent represents the fractal degree and can be interpreted as a "biomarker" of relative diseases. The previous study showed that the average wavelet method provides the most accurate results to estimate this scaling exponent when applied to stride interval time series. The purpose of this paper is to determine the most suitable mother wavelet for the average wavelet method. This paper presents a comparative numerical analysis of 16 mother wavelets using simulated and real fractal signals. Simulated fractal signals were generated under varying signal lengths and scaling exponents that indicate a range of physiologically conceivable fractal signals. The five candidates were chosen due to their good performance on the mean square error test for both short and long signals. Next, we comparatively analyzed these five mother wavelets for physiologically relevant stride time series lengths. Our analysis showed that the symlet 2 mother wavelet provides a low mean square error and low variance for long time intervals and relatively low errors for short signal lengths. It can be considered as the most suitable mother function without the burden of considering the signal length.

1. Introduction

Walking is the most common mode of human movement [1] and human gait involves the complex interactions of many nonlinear systems [2]. Disease, aging, trauma and genetic disorders can all have significant effects on human gait [3–5]. The locomotor system is a functional composite system of a number of body systems. The composite system is the integration of contributions from the central nervous, musculoskeletal, cardiopulmonary and metabolic systems. Specific central nervous system input from the cerebellum, motor and premotor cortices, and the basal ganglia, as well as peripheral feedback from visual, vestibular and proprioceptive sensors lead to adjustments and adaptations of locomotor system function relative to internal and external conditions [2,6]. In a healthy subject, the stride intervals display long-range power-law correlation, which may be the result of peripheral input or lower motorneuron control, or of the walking rhythm as controlled by higher nervous system centers [7].

Stride interval times series possess complex statistical properties [6,7,4,8]. Variance in healthy stride interval is not truly random and possesses temporal structure from one stride to the next [7,4]. Stride interval demonstrates fractal characteristics meaning that the stride to stride variations over a few strides are similar to those over hundreds of strides [4,9,10]. Fractal processes can be used to describe the natural irregularity of gait process because of temporal correlation [8].

Neurophysiological changes may alter the stride–interval correlations. Advanced age is an example of one condition known to affect neurophysiology and impact gait. These effects include decreased nerve conduction velocity, loss of motoneurons, decreased reflexes, reduced muscle strength, decreased proprioception and reduced central processing capabilities [7]. Parkinson's disease and Huntington's disease are neurodegenerative disorders of the central nervous system that produce pathological changes in the basal ganglia. These changes in basal ganglia may be associated with diminished stride–interval correlations associated with these disorders [7]. Amyotrophic lateral sclerosis affects the motoneurons of the cerebral cortex, brain stem, and spinal cord, and has been related to changes in walking speed [4]. A commonality seen in all of these neurodegenerative conditions and diseases has been an increase in the stride interval time and the magnitude of fluctuation as related to the fractal [7,4].

Many studies have showed the fractal properties of stride interval time series in different pathologies such as amyotrophic lateral sclerosis [4], Huntington's disease [4,7], and Parkinson's disease

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[11,12]. Most of studies are based on detrended fluctuation analysis (DFA). For example, the presence of fractal properties in gait during auditory cues was revealed using the DFA method [13]. In [14], authors controlled the movements of subjects and reinterpreted DFA of minimizing the stride-to-stride variations in walking speed. Certain recent works focus on using machine learning techniques, such as support vector machine and hidden Markov model, for different gait patterns recognition and disease diagnosis [15–19]. Wavelet-based signal projection was also applied for classifying gait signals of certain neurological disorders [20]. Some other works in this direction differentiated patients with neurodegenerative diseases using phase synchronization and conditional entropy [21].

It has been shown that stride intervals represent a stochastic process with power spectral density equal to $S(f) = \frac{C_\omega}{|f|^\beta}$ [22]. β value can be estimated in the time domain using techniques such as dispersional analysis, bridge detrended scaled window variance and detrended fluctuation analysis [23,24], in the frequency domain by using the ${}^{low}PSE_{we}$ method [25], and in the time-scale domain with the average wavelet coefficient (AWC) method [26]. Previous contributions have shown that the AWC method shows a uniform performance for the range of fGn and fBm class signals [6].

The purpose of this paper is to determine the most suitable mother wavelet for the average wavelet coefficient method to accurately estimate the β exponent in $1/f^\beta$ processes and to highlight the limitations of these mother wavelets. The tests include the $1/f^\beta$ process which is the best indicator of simulated signals of physiological processes. To achieve our goal, we used simulated $1/f^\beta$ processes with characteristics similar to stride interval time series, and we also used stride interval time series obtained from several patient groups [27].

2. Methodology

2.1. $1/f^\beta$ Processes

The $1/f^\beta$ processes are statistically self-similar random processes that generally have an inverse power relationship between measured power spectra and frequency $S(f) = 1/f^\beta$ [27–33]. In general, the $1/f^\beta$ processes are classified into two different models: fractional Brownian motion (fBm) and fractional Gaussian noise (fGn), as proposed by Kolmogorov [27,33,34]. Fractional Brownian motions are processes corresponding to $1 < \beta < 3$, where $\beta = 2$ represent the Wienner process, which entails classical Brownian motion. The fractal Gaussian noise represents processes corresponding to $-1 < \beta < 1$, while $\beta = 0$ is stationary white Gaussian noise that has a flat spectrum [27,33]. The two models are degenerated at the boundary where $\beta = -1$, $\beta = 1$, $\beta = 3$ [33]. The fractional Brownian motion process is nonstationary with stationary increments, while the Gaussian noise process is stationary [27,28,33]. The probability distribution of fGn signal is independent of segment length and position [27]. For the fBm signal, the probability distribution in the short sampled segment is equal to the long segment when the long segment is rescaled [27]. There is infinite low-frequency power for fractional Brownian motion that has finite power in any finite time interval and for the fractional Gaussian noise there is infinite high-frequency power [33,35]. The cumulative summation of the nonstationary fBn signals results in fGn signals [27,33,36].

The Hurst exponent can be estimated by the average wavelet coefficient method. The Hurst exponent is a parameter related to the fractal dimension, which represents the smoothness of a time series [37]. The relationship between Hurst exponent and the fractal dimension is given as follows: $D = 2 - H$, where the range of H is $0 < H < 1$. The relationship between each class's Hurst exponent and $1/f^\beta$ is [38]:

$$H_{fGn} = \frac{\beta + 1}{2} H_{fBm} = \frac{\beta - 1}{2} \tag{1}$$

The Hurst exponent can be denoted as H_{fGn} and H_{fBm}, and the processes differ in significant ways corresponding to $0 \le H \le 0.5$, H=0.5, and $0 \le H \le 1$ [34]. H=0.5 is the special case, here $H_{fGn} = 0.5$ means white Gaussian noise and $H_{fBm} = 0.5$ is Brownian motion. $H_{fGn} < 0.5$ is anti-correlated Gaussian noise while $H_{fGn} > 0.5$ is correlated noise [28]. $H_{fBm} < 0.5$ is anti-persistent Brownian motion while $H_{fBm} < 0.5$ is persistent Brownian motion [28]. From the equations we gather that $\beta = 0$ is white Gaussian noise, $\beta = 1$ is pink noise, and $\beta = 2$ is Brownian motion.

The concept of fractals can be used to model certain aspects of physiological dynamics such as fractal lungs, blood pressure, walking and hearts [7,23,39–41]. Healthy heart rate, blood pressure and walking produce pink noise in the output signal. These noises lose their $1/f$ characteristics due to age and disease, becoming either white or Brownian [22,42,43]. For example, the gait of patients with Parkinson's disease has stochastic behavior similar to the Brownian process [22,44].

2.2. Estimating β values via the average wavelet coefficient method

The wavelet transform maps the time domain waveforms into a scale-–time domain and estimates the signals both in the both time and scale domains [45,56]. The main idea of wavelet transform is to compare the similarity between an original waveform and the basic function called mother wavelet. The calculation of a process generates a dilated and translated version of the mother wavelet and the process of calculations performed at every scale and time point represents the continuous wavelet transform, which is usually used to study the fractal process [5]. The wavelet is denoted by a scale parameter a and a translation parameter b, where a is a positive number and b is a real number. The wavelet basis can be given by a single function called mother function [46]:

$$\psi_{a;b}(x) = \psi\left(\frac{x - b}{a}\right) \tag{2}$$

The continuous wavelet transform is given as

$$W[h](a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \psi^*_{a;b}(x) h(x) dx \tag{3}$$

Mother wavelet represents the basic function for the wavelet transform as does the sine in the Fourier transform. Fig. 1 illustrates some commonly used mother functions in our experiments [47].

The average wavelet coefficient method is used to estimate the Hurst exponents. To find the Hurst transform, the data is transformed into the wavelet domain by using wavelet and the arithmetic mean is used to calculate the averaged wavelet coefficient with the following

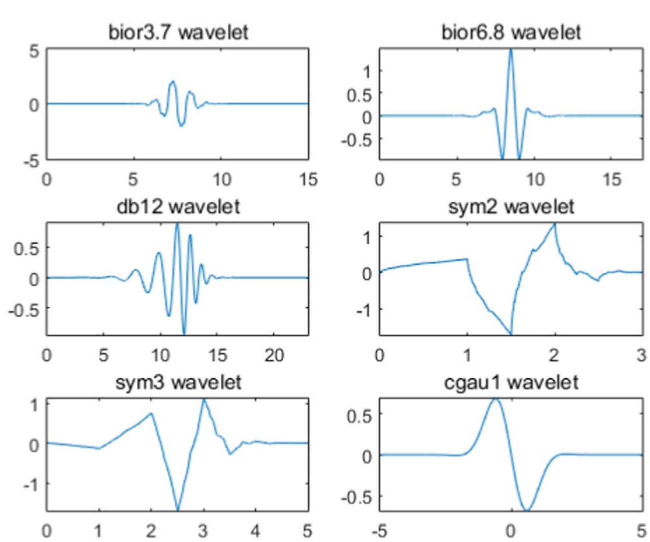

Fig. 1. Example mother wavelets used in our experiments.