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Fractal and multifractal characterization of the scaling geometry of an urban bus-transport network

P. Pavón-Domínguez [a,*], A.B. Ariza-Villaverde [b], A. Rincón-Casado [a], E. Gutiérrez de Ravé [b], F.J. Jiménez-Hornero [b]

[a] Dept. of Mechanical Engineering and Industrial Design, School of Engineering, University of Cádiz, Spain
[b] Dept. of Graphics Engineering and Geomatics, School of Engineering, University of Córdoba, Spain

1. Introduction

Fractal behaviour of networks has drawn the interest of researchers in many fields owing to the fact that it provides a framework for the treatment of irregular and seemingly complex shapes displaying similar patterns over a certain range of scales (Tarboton, Bras, & Rodríguez-Iturbe, 1988). These authors demonstrated that river networks can be viewed as fractal measures, and their fractality was quantified through the fractal dimension. Indeed, one of the properties of networks is self-similarity, which is commonly related to fractality. Through the fractal dimension, fractal objects are defined as a measure of complexity (Feder, 1988). The fractal dimension is a non-integer number that quantifies the density of the fractal in the metric space, and it is commonly used as a tool to identify the degree of complexity of a fractal, allowing comparison with another fractal (Mandelbrot, 1982; Schroeder, 2009; Tricot, 1995). Moreover, when the complexity of the spatial distribution of networks cannot be properly characterized by a single fractal dimension, multifractal analysis is required (Grassberger & Procaccia, 1983; Halsey, Jensen, Kadanoff, Procaccia, & Shraiman, 1986). Multifractal analysis can be understood as a generalization of monofractal analysis by the addition of information about the relative intensity in each place of the phenomenon under consideration (Sémécurbe, Tannier, & Roux, 2016). Multifractal analysis provides a distribution of singularities adequately describing both the heterogeneity of fractal patterns and the statistical distribution of measurements across spatial scales.

Both fractal and multifractal approaches have also been widely applied to characterizing natural networks as a useful tool to depict their spatial distribution and scaling properties in hydrology (De Bartolo, Gabriele, & Gaudio, 2000; Gaudio, De Bartolo, Primavera, Gabriele, & Veltri, 2006; Ijjasz-Vasquez, Rodriguez-Iturbe, & Bras, 1992; Rinaldo, Rodriguez-Iturbe, Rigo, Ijajasz-Vasquez, & Bras, 1993; Saa, Gascó, Grau, Antón, & Tarquis, 2007; Schuller, Rao, & Jeong, 2001; Veltri, Veltri, & Maiolo, 1996), medicine (Schmoll et al., 2011; Zhang, Liu, Dean, Sahgal, & Yue, 2006), biochemistry (Cheng, Dong, & Wang, 2005; Tang & Marangoni, 2006), geophysics (Berkowitz & Hadad, 1997), and manufacturing (El-Sonbaty, Khashaba, Selmy, & Ali, 2008), among other. Moreover, Batty and Longley (1994) stated that urban patterns can be understood as fractal structures. According to this, urban networks (streets, transportation systems, built-up spaces) have been widely studied from a fractal approach (Batty, 2008; Benguigui, 1995; Feng & Chen, 2010; Frankhauser, 1998; Tannier & Thomas, 2013), among others. To date, scarce attention has been directed to the analysis of multifractality in spatial anthropogenic systems (Ariza-Villaverde, Jiménez-Hornero, & Guitérrez de Ravé, 2013; Meifeng Dai, Zhang, Li, & Wu, 2014; Murcio, Masucci, Arcaute, & Batty, 2015), such as urban transport networks, which feature direct human intervention and deliberate decision-making.

Urban transport networks operate on city streets. However, they cannot serve all streets for cost and accessibility reasons. Transport networks are designed to cover the city efficiently and to meet the asymmetrical demand of passenger movement. In addition, the degree distributions of their topologies are approximately given by a power law or exponential function, which is in agreement with Xu, Hu, Liu, and Liu (2007), whose research focused on bus transport in cities. Public transport systems in cities tend to share small-world properties and evince a strong degree-degree correlation that reveals their complex nature, including underground, railway or airline systems (Sienkiewicz & Holyst, 2005). Travel routes in rail and bus public transportation systems have also been studied from a complex weighted networks perspective. In the case of bus routes, the network appears to possess a topological hierarchy, with a clustering spectrum decreasing according to a power law (Soh et al., 2010). All these findings might indicate subjacent (multi-)fractal behaviour in bus networks. In this study, an urban bus transport network is chosen due to its adaptability in the short and medium term, as opposed to street planning and roads. Bus transport systems require less infrastructure investment than tramway, underground and local train. In addition, they can be modified easily and quickly to adapt to changes in the city and urban development morphology.

Complex networks have also been demonstrated to exhibit self-similarity properties (Song, Havlin, & Makse, 2005), involving the fractal theory. One widely-employed method for fractal analysis of complex networks is the so-called box-covering algorithm for complex networks (Song, Gallos, Havlin, & Makse, 2007). These authors compared several box covering algorithms by applying them to a number of model and real-world networks. For these networks, box size is given in terms of network distance, which corresponds to the number of edges on the shortest path between two nodes (Wei et al., 2013). Simultaneously,

* Corresponding author at: Escuela Superior de Ingeniería, Avenida de la Universidad de Cádiz, 11519 Puerto Real, Cádiz, Spain.
E-mail address: pablo.pavon@uca.es (P. Pavón-Domínguez).

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multifractal analysis of unweighted complex networks has drawn growing interest in recent years (e.g. Furuya & Yakubo, 2011; Wang, Yu, & Anh, 2012). Meanwhile, Liu, Yu, and Anh (2015) first introduced the sandbox algorithm to explore the multifractal behaviour of unweighted complex networks, demonstrating that this is the most effective, feasible and accurate method to estimate multifractal parameters. Yet, due to the fact that edge-weights values in weighted networks could be any real numbers excluding zero, fractal and multifractal approaches were completely unfeasible for weighted networks. To overcome this drawback, Wei et al. (2013) developed a box-covering algorithm to determine the fractal dimension of weighted networks, and recently, an extension of the sandbox algorithm has been developed by Song, Liu, Yu, and Li (2015). The sandbox method was first introduced by Tél, Fülöp, & Vicsek, 1989 and was later developed by Vicsek (1990) and Vicsek, Family, and Meakin (1990). De Bartolo, Gaudio, and Gabriele (2004) generalized the sandbox method for river networks, highlighting that this method adapts to negative moment orders and solves border effects. It is based on a covering succession of radius R circles whose centre is randomly distributed in the fractal. The main advantage of this method is related to the high number of large boxes obtained and thus, the greater capacity to accurately estimate multifractal parameters, mainly for negative moment orders, allowing for the reconstruction of a more accurate multifractal spectrum (Lopes & Betrouni, 2009).

In this paper, we hypothesize that urban transport networks, as well as others existing in nature, might exhibit a multifractal behaviour. Verification of this hypothesis allows us to include their multifractal nature as another determining factor in designing urban transport networks. The (multi-)fractal exploration is conducted from two perspectives. First, in terms of its geometry by means of the traditional box-counting fractal analysis, and second, by using the recent extension of the sandbox algorithm for multifractals conducted by Song et al. (2015). The novelty of the application of this multifractal approach is the multifractal study of a bus network as a weighted graph, in which nodes are bus stops and edges are the distance between them.

2. Materials and methods

2.1. Urban bus-transport network

In selecting the urban bus-transport network under study, priority was given to: i) mobility in the city being conditional on the exclusivity of a bus transportation system, without alternative public transport, such as underground, tramway or local trains, available. ii) Absence of large populations surrounding the city. Cordoba (37°85′N; 4°85′W) is one of the most populated cities in Spain (327,362 inhabitants), and its urban public transport system relies solely on its bus network. Given the dependence of mobility in the city on the bus transport network, urban bus-transport in Cordoba was selected for this study.

The bus network is managed by a municipal company (AUCORSA), which operates several bus lines classified into three types: urban (Fig. 1), outskirts and temporary services. Meanwhile, outskirts routes connect the city centre with several dependent villages, and temporary services operate when required for social and sport events. As the purpose of this study is to analyse regular mobility within the city, neither outskirts nor temporary services were taken into consideration. Urban routes are comprised by 15 bus lines, which were selected for this study. Fig. 2 exhibits each bus route (black line) over the complete bus network (grey line). Every urban line follows a circular route that starts and finishes at a fixed bus stop. In general, urban lines start out from a bus stop in a peripheral district and return to the same starting point having run through the city centre, where most bus stops are shared among bus lines. This being the case of lines 1, 2, 3, 4, 5, 6, 7, 8, 9 and 12. Line 14 is similar to previous ones, but it does not go through the city centre. Another type of line route connects the city centre with a peripheral district and returns to the starting point, such as lines 10, 11 and

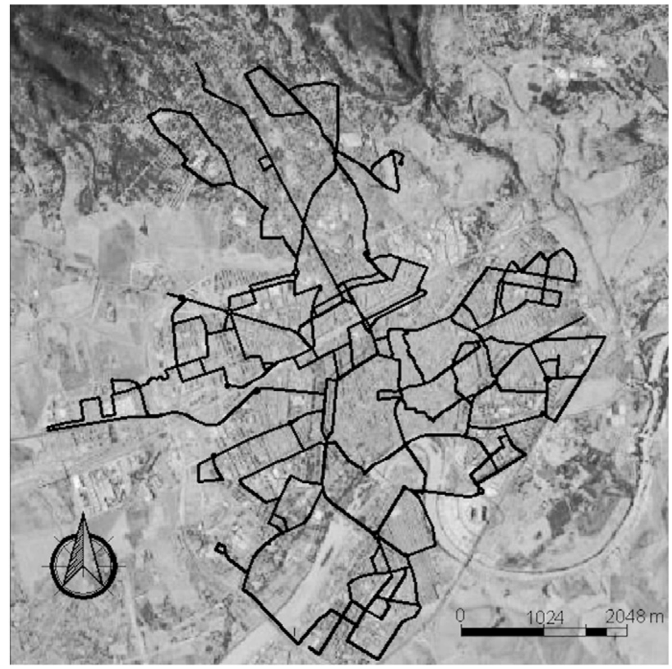

Fig. 1. Urban bus network of Cordoba.

13. The latter, which is identified as C2, is served by electrical microbuses and runs along the right side of the historic district of the city where motor vehicle traffic is restricted. C2 is characterized by a lack of bus stops along its route. Table 1 summarizes geometrical characteristics of the urban bus lines.

The total area considered was a square with a side of 8192 m in length, resulting in a 67.11 km^2 map. The urban bus network of Cordoba is included in a square 4 m resolution image, with each pixel representing a 16 m^2 area. This image was subsequently transformed into a binary matrix, in which pixels containing bus routes take the value of 1 and the remaining pixels take that of 0. The mass distribution of pixels containing line routes was analysed by applying the fractal analysis. Furthermore, the bus network was represented as a set of nodes (bus stops) and edges (distance between consecutive nodes) in order to apply the modified sandbox algorithm for weighted networks (Song et al., 2015).

2.2. Fractal analysis

Fractal dimensions were calculated by applying the traditional box-counting method (Mandelbrot, 1982). This procedure consists of covering the image with different square boxes, which increase the size, r, in successive steps, and later counting the amount of boxes, $N(r)$, which are required for completely covering the network. Thus, r and $N(r)$ are related in a double logarithmic plot in order to determine the fractal dimension of the network. Where both variables follow a linear relationship between a specific spatial range of scales, $r_{\min}$ and $r_{\max}$, the fractal nature of the network can be trusted. According to the equation:

$$D = \lim_{r \to 0} \frac{\ln N(r)}{\ln(1/r)} \quad (1)$$

the fractal dimension, D, is estimated from the slope of the linear segment in the previously defined range.

The choice of an appropriate scaling range is widely known to be a crucial step, and it must be executed before conducting the fractal analysis. Selected inner and outer scales usually have a significant effect on fractal dimensions (Saucier & Muller, 1998). In this regard, configuration entropy explores the effect of scale on any measure defined on a