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Form-finding algorithm for masonry arches subjected to in-plane earthquake loading

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ABSTRACT

This paper presents the first form finding method for masonry arches subjected to self-weight and in-plane horizontal loading due to earthquakes. New material-efficient arch shapes are obtained by considering both horizontal and gravitational acceleration in the form finding process. By interpreting the obtained forms, insights into the influence of form on the earthquake resistance of the arches are presented. The form finding algorithm relies on two simplified, first-order equilibrium methods: thrust line analysis and kinematic limit state analysis, which present respectively a lower- and upper-bound approach to the analytic problem of arch stability under gravity and horizontal loading. Through a methodological application of a series of geometric manipulations of the thrust line, shapes are obtained that can resist the design acceleration by guaranteeing a compression-only load path. Forms are obtained for horizontal accelerations of 0.15, 0.3 and 0.45*g*, as well as for arches of different rise-to-span ratios (1/2, 1/4 and 1/8). The obtained shapes require up to 65% less material than circular arches with constant thickness that are designed to withstand the same horizontal acceleration and self-weight, regardless of acceleration magnitude. The findings of this research will thus allow more material-efficient design of masonry arches in seismic areas.

1. Introduction

Masonry arches have been the subject of extensive research ever since Robert Hooke published his findings that "*the true mathematical and mechanical form of arches*", is the inverse of a hanging chain [1]. Most ensuing research, however, has focused on the analysis of existing masonry arches, rather than on how to shape these types of arches appropriately for specific loading conditions. In particular, literature has not yet addressed the question of finding the appropriate form for masonry arches under earthquake loading. Therefore, this paper presents a new form finding algorithm that allows for the design of arches that can better withstand earthquakes while also employing less material than traditional circular and catenary arch shapes.

1.1. Form finding of arches

To the best of the author's knowledge, the finding of appropriate shapes for masonry arches under horizontal loading has to date not been reported in literature, whereas the finding of appropriate shapes under gravity loading has been extensively researched. After Hooke's seminal work, a panoply of authors expanded on his findings, focusing on analyses techniques that could help shape arches. De La Hire's work on thrust lines [2] and Coulomb's studies on hinge formation and sliding in arches [3] were both milestones that helped pave the way for the extensive treatises on masonry bridge design in the 18th and 19th centuries. For an extensive overview of the evolution of arch design the reader is referred to [4,5]. All of this research, however, was focused on the design and construction of arches subjected to gravity loading. Even after renewed interest in masonry arches was spurred by Heyman's work in the 1960s [6,7], the research focus remained on how to analyze and optimize these arches under vertical loading. Indeed, several authors tackled the problem of the optimal arch under vertical loads through analytical and numerical approaches [8–10]. Peng carried out an interesting study employing limit state analysis in combination with a genetic algorithm to find the form of arches [11] and shape optimization tools were used to design concrete [12] and steel arches in the context of bridge design. Another approach was developed to obtain the forms of spatial leaning arches, relying on the use of the thrust lines to create antifunicular forms [13]. Nevertheless, none of these recent studies address the form finding of arches when considering gravity and horizontal loading. Uzman et al. presented a method to optimize the design

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of parabolic and circular arches with varying cross section under a variety of loads, but their method does not allow to account for several load cases [14]. Therefore, it does not cope with the horizontal forces that can occur in both in-plane directions during a seismic event. Therefore, this paper relies on the extensive work of several authors on the analysis of arches under earthquake loading, to inform form finding procedures for arches that can better resist such loads. An overview of this body of work is given in Section 1.2.

1.2. Review of analysis techniques for masonry arches under earthquake loading

The analysis of masonry arches under earthquake loading, in contrast to their form finding, has been approached through a variety of methods. One method, thrust line analysis, is an equilibrium method that hinges on Hooke's hanging chain analogy. It relies on three assumptions that are generally accepted for masonry: (1) masonry has no tensile strength; (2) sliding between blocks does not occur; and (3) the compressive stresses remain low compared to the material strength of the masonry so that crushing does not occur. If these assumptions hold, Heyman's safe theorem states that an arch will be stable if a thrust line can be found that fits within the geometry of the arch under the considered loading [6]. While only static loads are considered in this analysis method, DeJong et al. and Huerta have simulated equivalent earthquake loading by imposing a horizontal load to arches by tilting them [15,16]. This approach induces a horizontal acceleration, but also reduces the compressive stresses under gravity. However, as crushing is assumed not to occur, these reduced compressive stresses can be ignored [15], and thus an arch will be stable if a thrust line can be found that fits within the masonry under the combination of the considered horizontal acceleration and gravity. Thrust line concepts have been further expanded to 3D-networks in the thrust network analysis, which allows for the structural design and analysis of arches and shells under vertical loading [17,18]. Relying on the same principles, other authors have developed other continuum approaches to perform lower bound equilibrium analysis looking at thrust surfaces in vaults and shells [19]. One of the strengths of the thrust network analysis method, however, is that it uses a dual approach, visualizing the forces in the eventual form using graphic statics by employing force network polygons. This method was reformulated and extended neglecting this duality with graphic statics to be able to cope with horizontal forces, for example due to earthquakes [20] and sample application to arches was provided. Overall, the thrust line analysis approach is a lower bound (static) solution to the problem of stability: every thrust line that can be found within the masonry arch, represents one possible equilibrium solution between the internal and external loads [21]. Therefore, it is necessary to find the maximum acceleration under which a thrust line can fit within the arch's geometry, which can be done through linear programming [22]. This thrust line also provides a qualitative idea about the expected collapse mechanism of the arch, as the locations where the thrust line is closest to the boundary are the locations where hinges for this mechanism are expected to form.

Another equilibrium method that replaces the earthquake with an equivalent horizontal acceleration is based on the kinematic limit analysis of rigid blocks [23] and builds upon the same three assumptions as thrust line analysis. It furthermore relies on the observation that unreinforced masonry arches fail through a four-hinge mechanism under base motion [24]. It determines the critical collapse mechanism and associated acceleration needed to activate this mechanism through a series of virtual work calculations [25]. Kinematic limit analysis has been automated and applied to a set of different arch geometries with constant thickness and has been validated experimentally and numerically by several authors [26–29]. In this method, the acceleration required to activate every possible collapse mechanism is determined. The lowest acceleration that triggers a collapse mechanism corresponds to the critical acceleration and determines the capacity of the arch. This kinematic limit state approach is therefore an upper bound solution to the same problem solved by thrust line analysis [21]. Upper bound limit state analysis has also been expanded to analyze three-dimensional vaults [30,31] and has been applied in the context of seismic loading [32].

Both thrust line analysis as limit state analysis replace the dynamic earthquake loading by an equivalent horizontal load and are essentially stability analyses. They can predict the minimum acceleration required to onset the formation of the critical mechanism of the arch [33], but cannot account for what happens once the mechanism is activated under base motion. Once the critical mechanism is activated, the blocks that have formed will start rocking back and forth. The question whether the arch collapses or not, becomes a problem of rigid body dynamics. The dynamic response of arches through these continuous cycles of rocking can then be determined analytically or numerically by solving the equation of motion for a single-degree-of-freedom (SDOF) system [33,34]. However, if the acceleration to onset the mechanism is not exceeded, the rocking will not occur. Therefore, both thrust line analysis and limit state analysis yield conservative solutions.

Other approaches used to assess the behavior of masonry arches during earthquakes include non-linear finite element modeling (FEM), and distinct element modeling (DEM). Despite the widespread use of FEM for structural analysis, the application of FEM to masonry remains a convoluted task. Because of its discontinuous nature, masonry cannot be modeled as an elastic continuum [27] and therefore, computationally expensive non-linear analyses are necessary. These non-linear finite element models have nevertheless been applied successfully to masonry arch bridges [35] and seismic analyses were performed through nonlinear static pushover simulations [36,37] and nonlinear dynamic analysis, which provided similar results [38]. Non-linear FEM requires high expertise from its operators, but has the advantage of being able to capture three-dimensional effects. DEM can similarly capture these three-dimensional effects and inherently incorporates the heterogeneous nature of masonry. This method relies on finite-differences principles to characterize the interaction of discontinuous blocks. Blocks are allowed to rotate and displace, deform and form new contacts and movements are traced at each time-step allowing the user to understand the collapse of the arch over time [15,39]. This method has been applied to arches under seismic action [27,33,34] and its results were found to correspond well with data obtained through analytical limit state analysis [15,27] and experimental tests [15].

The convoluted nature of non-linear FEM and DEM, and their high computational demands are in stark contrast to the straightforward analytical solutions that can be obtained employing thrust line and limit state analysis under an equivalent horizontal load. This observation makes these latter methods (thrust line and limit state analysis) more suitable for the design and form finding of masonry arches under earthquake loading, especially as these methods have been shown to be accurate and conservative [15,22].

2. Form finding methodology

2.1. Thrust lines due to combination of gravity and horizontal acceleration

The form finding method presented in this paper relies on thrust line analysis performed under combined vertical and hori-