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# Identification and quantification of spatial interval uncertainty in numerical models

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ABSTRACT

This paper presents a novel methodology for the identification and quantification of spatial uncertainty, modelled as an interval field. In order to make a realistic assessment of the spatial uncertainty on the model parameters, the dimensionality of the interval field as well as its constituting base functions and interval scalars have to be identified. For this purpose, this work introduces an identification method based on objective measurement data. The specific challenge in this context lies in the fact that a continuous spatial input parameter has to be identified on a high-resolution discretised model of the structure under consideration, based on possibly high-dimensional measurement data set, obtained in the result domain of the analysed model. In the presented method, the field dimensionality is quantified based on the concept of effective dimension of the measurement data. The base functions of the interval field are identified by minimising the difference between the gradients of the halfspaces respectively bounding the measurement data and the realisations of the interval field. The method is illustrated using two case studies: an dynamic model of a cantilever beam and a quasi-static model of a cast pressure vessel. It is shown that the presented methods are capable of accurately identifying the interval field uncertainty that is present on the model parameters, and that this identification is robust against the size of the measurement data set.



## 1. Introduction

Continuing advances in computing power and the upswing of high performance computing (HPC) possibilities allow for making highly detailed numerical models to approximate the partial differential equations that govern most physical processes. This in its turn enables informed design choices based on numerical datasets, even in very early stages in a product development cycle. However, such a deterministic analysis may not be sufficient to assess the quality of a design, as depending on the design stage, some physical properties of the model are unknown or not yet determined. Moreover, even in a final design stage production tolerances, uncertainty about the loading situation and natural variability in material properties often introduce significant non-determinism in the functional behaviour of the design. Recent approaches in the field of computational engineering therefore aim to incorporate model non-determinism into numerical design models in either a parametric [1] or non-parametric way [2]. As such, a realistic assessment of the reliability of the design, including the different sources of non-determinism is obtained. Moreover, robustness of the design with respect to these variations can also be ensured. In this context, two supplementary philosophies exist: probabilistic and possibilistic numerical analysis. Both techniques have their own field of applicability, depending on the amount of information is available to the designer [3].

As a complement to this well-established framework of probabilistic uncertainty representation, possibilistic techniques such as Interval FE (IFE) or Fuzzy FE (FFE) were introduced. Following these techniques, the non-determinism is respectively depicted as an interval or fuzzy set and thus propagated through the numerical model. These concepts eliminate the need for the identification of a full probabilistic data description, which may be very cumbersome. Moreover, less expensive numerical procedures are necessary for the description of the non-determinism [4–7], which makes these techniques highly suitable for early design stages. Interval fields were only recently introduced as an extension to this concept to account for non-deterministic parameters that have a non-homogeneous distribution over the model domain. Interval Fields can be regarded as a possibilistic counterpart to the established framework of Random Fields [8–11] (see e.g., [8] for a description on Random Fields). The description of an interval field is based on the superposition of $n_b$ base functions $\psi_i$, scaled by independent interval scalars $\alpha_i^I$. The base functions $\psi_i$ describe

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the spatial nature of the non-deterministic value that is modelled by the interval field over the model domain, and are unit-less. The interval scalars $\alpha_i^I$ on the other hand quantify the non-determinism of the model parameters under consideration. An interval field is formally expressed as:

$$\gamma_F^I(\mathbf{r}) = \sum_{i=1}^{n_b} \psi_i(\mathbf{r})\alpha_i^I \quad (1)$$

Application of these powerful interval techniques for obtaining a robust and reliable design requires the identification of their driving parameters. In the context of quantifying scalar interval uncertainty, to date most presented methods employ a squared $\mathcal{L}_2$-norm based objective function that is aimed at minimising the discrepancy of the separate interval boundaries of respectively a measurement data set and the prediction of the interval FE model (see e.g. [12–15]). An alternative method was introduced by Khodaparast et al. who used a Kriging predictor for the inverse propagation of deterministic measurement points in order to estimate the hypercubic interval uncertainty on the input parameters of the model under consideration [16]. The authors recently proposed a generic methodology for the identification and quantification of multivariate interval uncertainty in [17,18]. The methodology is build on the concept of using convex polytopes to bound the non-determinism in both the set of repeated measurement data and the result of propagating the non-deterministic parameters through the numerical model under consideration. Minimisation of the discrepancy between both convex polytopes provides the interval non-determinism in the model parameters corresponding to an optimal model description of the non-determinism that was present in the measurement data. This methodology was validated using simulated measurement data and was proven to yield highly accurate results at limited computational cost.

However the good performance of these novel techniques, they are all limited to interval uncertainty that is considered to be continuous over the model domain, which in a number of cases is a serious underestimation of the spatial complexity of the non-determinism. Therefore, it as yet remains unclear how parametric non-homogeneous interval non-determinism, modelled as a continuous interval field, can efficiently be identified based on experimentally obtained datasets of system responses. Specifically, the base functions $\psi_i$ and field dimensionality $n_b$, as defined in Eq. (1) should be quantified in order to make an accurate and truthful estimation of the spatial complexity of the interval field non-determinism under consideration. An initial estimation of the base functions and field dimensionality can be based on expert knowledge. This expert knowledge might stem from previous experiments, knowledge of the manufacturing process, or just operator experience. In most realistic industrial design cases however, this expert knowledge is scarce, ambiguous or too subjective for use in the context of designing reliable end-use components. In order to obtain an objective quantification of the uncertainty that is present in the design model, this initial estimate has to be improved and/or validated based on experimentally obtained measurement data. However, how this should be done in an interval context as yet remains unclear. The main challenge in this context is that the continuous interval field has to be defined over a discretised domain, possibly having a high spatial resolution. Moreover, in a realistic FE model, a large number of responses can be considered, which have to be compared to measurement data. This paper therefore introduces a method for identifying spatial uncertainty, modelled as an interval field, based on a large set of measurement data. In this context, both the dimensionality of the problem at the input and output side of the numerical model under consideration is reduced. A generic procedure is introduced for the identification and quantification of both the field dimensionality $n_b$ and the base functions $\psi_i$. Moreover, in order to limit the computational burden of the identification using high-dimensional datasets, the dimensionality of both the measurement data set, as well as the uncertain set containing the responses of the interval field FE model, are reduced based on a singular value decomposition of the covariance structure of the measurement data set. The methodology is illustrated on two case studies using numerically generated measurement data. It is shown that an accurate identification of the complete interval field is obtained following the proposed procedure.

Section 2 introduces the interval field finite element method in general. The identification of the interval scalars, as proposed by the authors in [17,18], is explained in Section 3. The novel methodology for the identification of the spatial topology of the interval field, as well as the reduction scheme, are introduced in Section 4. Sections 5 and 6 illustrate the methodology in two case studies. Finally, the most important conclusions are given in Section 7.

## 2. The interval field finite element method

This section provides a summary of the interval field finite element method for the estimation of the uncertainty in the responses of a numerical model containing interval field uncertainty on its model parameters. The method is presented in a generic sense, and the most important definitions are listed. By definition, an interval parameter $x$ is indicated using apex I: $x^I$. Vectors are expressed as lower-case boldface characters $\mathbf{x}$, whereas matrices are expressed as upper-case boldface characters $\mathbf{X}$. For the remainder of the text, interval parameters are either represented using the bounds of the interval $x^I = [\underline{x}; \overline{x}]$ or the centre point $\mu_{x^I} = \frac{\underline{x}+\overline{x}}{2}$ and interval radius $\Delta x^I = \frac{\overline{x}-\underline{x}}{2}$.

### 2.1. Interval field computations

Consider a numerical model $\mathcal{M}(\gamma)$, parametrised by parameter vector $\gamma \in \mathbb{R}^k$, which includes e.g. material stiffness or plate thickness values. This model $\mathcal{M}(\gamma)$ translates $\gamma$ to a vector of responses $\mathbf{y} \in \mathbb{R}^d$ (e.g. resonance frequencies or stresses) through the function operator $g$, which is defined as:

$$\mathcal{M}(\gamma) : \mathbf{y} = g(\gamma), \quad g : \mathbb{R}^k \mapsto \mathbb{R}^d \quad (2)$$

Usually, $g$ is the finite element model that is used to approximate the solution of the system of partial differential equations that are used to model the design model behaviour. In the specific case when dynamic design problems are considered, $\mathbf{y}$ usually consists of the eigenfrequencies and eigenmodes of the system at hand. The interval field FE method can then be expressed as finding the uncertain set of system responses (i.e. the solution set) $\tilde{\mathbf{y}}$, when the model parameter uncertainty is depicted as an interval field $\gamma_F^I(\mathbf{r}) \in \mathbb{IR}^k$ over the geometrical model domain $\Omega$, with $\mathbb{IR}^k$ the $k$-dimensional space of interval scalars and $\mathbf{r} \in \Omega \subset \mathbb{R}^t$. $t$ is the physical dimensionality of the problem at hand (e.g. $t = 4$ for a time dependent simulation in three physical dimensions). The solution set $\tilde{\mathbf{y}}$ usually spans a multidimensional manifold in $\mathbb{R}^d$. This manifold is in general not convex, which makes an exact computation numerically very hard. $\tilde{\mathbf{y}}$ is therefore commonly approximated by the construction of an uncertain realisation set $\tilde{\mathbf{y}}_s$, which is defined as:

$$\tilde{\mathbf{y}}_s = \left\{ \mathbf{y}_{sj} | \mathbf{y}_{sj} = g(\gamma_{F,j}(\mathbf{r})); \gamma_{F,j}(\mathbf{r}) \in \gamma_F^I(\mathbf{r}) \right\} \quad (3)$$