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# Multiphase topology optimization of lattice injection molds

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ABSTRACT

This work presents a topology optimization approach for lattice structures subjected to thermal and mechanical loads. The focus of this work is the design of injection molds. The proposed approach seeks to minimize the injection mold mass while satisfying constraints on mechanical and thermal performance. The optimal injection molds are characterized by a quasi-periodic distribution of lattice unit cells of variable relative density. The resulting lattice structures are suitable for additive manufacturing. The proposed structural optimization approach uses thermal and mechanical finite element analyses at two length scales: mesoscale and macroscale. At the mesoscale, lattice unit cells are utilized to obtain homogenized thermal and mechanical properties as a function of the lattice relative density. At the macroscale, the lattice unit cells are optimally distributed using the homogenized properties. The proposed design approach is demonstrated through 2D and 3D examples including the optimal design of an injection mold. The optimized injection mold is prototyped using additive manufacturing. The numerical model of the optimized mold shows that, with respect to a traditional solid mold design, a mass reduction of over 30% can be achieved with a small increase in nodal displacement (under 5 microns) and no difference in nodal temperature.



## 1. Introduction

During the injection molding cycle, injection molds are required to withstand pressure loads and thermal expansions while providing dimensional accuracy to the molded part. Molds are also required to uniformly transfer heat flux from the mold cavity, where the part is molded, to cooling channels filled with running coolant. While design guidelines are known to improve the thermal and mechanical performance of injection molds [1], the development of structural optimization methods such as topology optimization offers the potential to create novel and complex injection mold designs with higher performance [2].

With reference to heat conduction, topology optimization has been employed to minimize the temperature gradient magnitude distribution (heat dissipation) for thermal components including heat sinks for multichip modules [3] and thermal-fluid electronic microchannels [4]. Studies that consider coupled linear elasticity, heat conduction and the resulting thermoelastic load in topology optimization have been recently proposed for two-dimensional structures [5]. Thermal expansion has been considered in the topology optimization of micro-electro-mechanical-systems (MEMS) [6] and electronic packages [7]. Regardless of the numerical model, the result of the application of a traditional topology optimization algorithm is a solid-void structure in which intermediate densities are penalized. As an alternative to the solid-void structure, this work proposes the design of optimal injection molds with a lattice (porous) structure. The lattice structure uses intermediate densities, which relaxes the optimization problem, expands the design space, and, potentially, increases the performance of the final design [8]. In addition, a lattice injection mold opens possibilities for airflow assisted cooling and more complex heat exchanging design.

An optimal design of a lattice structure can be achieved through multiscale topology optimization. The multiscale topology optimization problem consists on finding the optimal lattice unit cell (LUC) designs (mesoscale structural optimization) as well as their optimal distribution in the structure (macroscale structural optimization). This method requires the application of asymptotic homogenization theory in order to derive the macroscale mechanical properties of the LUCs. The method has been effectively applied to 2D and 3D structures subjected to pure mechanical loads [9–12]. Despite of their potential, multiscale topology optimization has two main drawbacks. First, it requires the execution of several optimization problems in parallel for each iteration, which makes it computationally expensive, especially for 3D

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designs. Second, the solutions do not necessarily converge to a manufacturable, connected design and time-consuming post-processing may be needed. To alleviate the computational cost and to improve the connectivity of the LUCs, concurrent topology optimization strategies have been proposed [13–16]. These strategies lead to the design of an optimal macroscale structure composed of a periodic (uniformly distributed) LUC. Despite of their manufacturability, the LUC periodicity produces a suboptimal structure when compared to a non-periodic design.

An alternative approach is the use of multiphase topology optimization [17,18]. This approach uses pre-defined LUCs with homogenized mechanical properties avoiding the mesoscale structural optimization. The geometry of the LUCs is controlled by a few geometric parameters that define their relative density or porosity. Material interpolation schemes are defined to map the LUC relative density to their homogenized mechanical properties. The (macroscale) structural optimization problem consists on finding the optimal relative density distribution, which ultimately provides the optimal LUC distribution within the structure. Multiphase topology optimization has been applied to two-dimensional structures subjected to pure mechanical load [17,18]. The application of this approach to heat-transferring structures has been less reported in literature.

This work extends the use of multiphase topology optimization to three-dimensional structures composed quasi-periodic LUCs considering thermal and mechanical performance. Integral to the proposed multiphase topology optimization method for lattice injection molds is the use of a thermomechanical finite element model. In this model, the mechanical and thermal load analyses are coupled to predict the structure's thermoelastic response. Asymptotic homogenization theory is used to predict the isotropic thermal conductivity and the orthotropic linear elasticity of the LUCs.

The results are demonstrated with the topology optimization of lattice structures with minimum mass under elastic mechanical and thermal constraints, which include maximum nodal displacement and maximum nodal temperature on the mold cavity.

The remaining of the paper is organized as follows: The proposed design approach is presented in Section 2. The homogenization theory is explained in Section 3 and the macroscale structural optimization approach is explained in Section 4. Two numerical problems are presented to demonstrate the design approach in Section 5: (1) a 2D structure with thermal and mechanical loads, (2) a core of a 3D injection mold design. Finally, summary and conclusion are provided in Section 6.

## 2. Proposed multiphase topology optimization approach

The proposed optimization approach involves finite element models in two length-scales: mesoscale and macroscale. The mesoscale finite element models correspond to the lattice unit cells (LUCs). These models are used to predict the homogenized LUC properties. As a result, homogenized elastic and thermal coefficients are expressed as functions of the LUC relative density. The macroscale finite element model corresponds to the injection mold. This model contains mechanical and thermal boundary conditions, which include external mechanical loads and supports as well as the heat sources (mold cavity) and sinks (cooling channels). The macroscale design problem addressed in this work is to find the optimal distribution of given number of LUCs that minimizes the injection mold mass while satisfying mechanical and thermal constraints. These constraints include mechanical and thermal compliance as well as maximum nodal displacement and maximum nodal temperature.

The macroscale design problem is solved in two steps: First, a relaxed convex problem is addressed so that the mass is minimized subject to constraints on mechanical compliance and a thermal compliance [19]. The result is a global optimum of a convex problem to be used as the initial design of a non-convex problem. Second, using this initial design, a structural optimization algorithm finds the optimal distribution of a discrete number of LUCs so that the maximum displacement and temperature are minimized in specific locations of the injection mold, e.g., mold cavity. The optimization approach is summarized in Fig. 1.

## 3. Mesoscale analysis and homogenization of elastic and thermal properties of lattice unit cells

This section summarizes the numerical approaches used to derive the homogenized elasticity tensor $\mathbf{D}_c^H$ and the homogenized thermal conductivity tensor $\boldsymbol{\kappa}_c^H$ of an a priori defined LUC. The theory presented in this section follows the principles of asymptotic homogenization [20–23].

### 3.1. Asymptotic homogenization of the elastic properties

Let a macroscale design domain $\Omega$ to be comprised of $n_c$ LUCs, where $c = 1, \ldots, n_c$. Each of LUC is further discretized into $n_e$ finite elements as illustrated in Fig. 2.

According to the homogenization theory for media with a periodic structure, the homogenized elasticity tensor $\mathbf{D}_c^H$ of a discretized periodic LUC is given by

$$\mathbf{D}_c^H = \frac{1}{|V_c|}\sum_{e=1}^{n_e}\int_{V_e}[\mathbf{I}-\mathbf{B}_e\boldsymbol{\chi}_e]^{\mathrm{T}}\mathbf{D}_e[\mathbf{I}-\mathbf{B}_e\boldsymbol{\chi}_e]\mathrm{d}V_e, \tag{1}$$

where $n_e$ are the number of finite elements of the discretized LUC, $|V_c|$is the LUC volume, $\mathbf{I}$ is the identity matrix, $V_e$ is the volume of the finite element $e, \mathbf{B}_e$ is the element strain-displacement matrix, $\mathbf{D}_e$ is the element elasticity tensor, and $\boldsymbol{\chi}_e$ is the matrix containing the element displacement vectors $\boldsymbol{\chi}_e^{ij}$ resulting from globally enforcing the unit test strains $\boldsymbol{\varepsilon}^{ij}$ (Fig. 3). For a 3D solid finite element, this is

$$\boldsymbol{\chi}_e = [\boldsymbol{\chi}_e^{11}, \boldsymbol{\chi}_e^{22}, \boldsymbol{\chi}_e^{33}, \boldsymbol{\chi}_e^{12}, \boldsymbol{\chi}_e^{23}, \boldsymbol{\chi}_e^{13}], \tag{2}$$

where $\boldsymbol{\chi}_e^{ij}$ are vectors of size $24 \times 1$. The element displacement vectors $\boldsymbol{\chi}_e^{ij}$ are obtained from the global displacement vector of the LUC $\boldsymbol{\chi}_c^{ij}$, which is the solution of the equilibrium equation

$$\left[\sum_{e=1}^{n_e}\int_{V_e}\mathbf{B}_e^{\mathrm{T}}\mathbf{D}_e\mathbf{B}_e\mathrm{d}V_e\right]\boldsymbol{\chi}_c^{ij} = \sum_{e=1}^{n_e}\int_{V_e}\mathbf{B}_e^{\mathrm{T}}\mathbf{D}_e\boldsymbol{\varepsilon}^{ij}\mathrm{d}V_e. \tag{3}$$

The first term in the left hand side of Eq. (3) is the stiffness matrix of the LUC and the right hand side is the nodal force vector of the LUC.

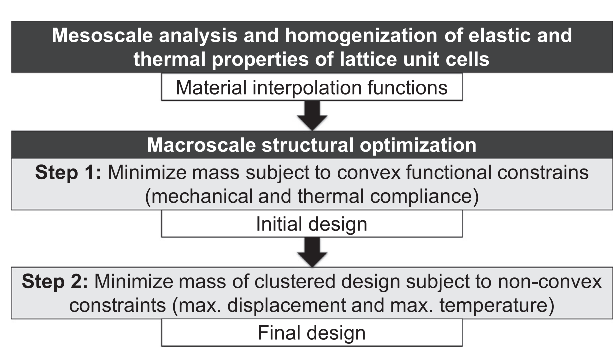


**Fig. 1.** Flowchart of proposed design approach.