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One-dimensional finite element formulation with node-dependent kinematics

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The present paper presents a refined one-dimensional finite element model with node-dependent kinematics. When this model is adopted, the beam theory can be different at each node of the same element. For instance, in the case of a 2-node beam element the Euler-Bernoulli theory could be used for node 1 and the Timoshenko beam theory could be used for node 2. Classical and higher-order refined models have been established with the Carrera Unified Formulation. Such a capability would allow the kinematic assumptions to be continuously varied along the beam axis, that is, no *ad hoc* mixing techniques such as the Arlequin method would be required. Different combinations of structural models have been proposed to account for different kinematic approximations of beams, and, beam models based on the Taylor and the Lagrange expansions have in particular been used. The numerical model has been assessed, and a number of applications to thin-walled structures have been proposed. The results have been compared with those obtained from uniform kinematic models and convergence analyses have been performed. The results show the efficiency of the proposed model. The high accuracy of refined one-dimensional models has been preserved while the computational costs have been reduced by using refined models only in those zones of the beam that require them.

1. Introduction

Improvements in the performances of next-generation structures will require the use of new computational tools that are able to deal with multi-field problems and to provide increasingly accurate results. Classical one-dimensional structural models are used widely in the design of complex structures but they are limited by their fundamental assumptions. When the Euler-Bernoulli [27] beam model is used, it is accepted that the solution can only be considered accurate for slender bodies and isotropic materials. If moderately stubby structures are considered the model proposed by Timoshenko [42] has to be used to include shear effects, and in this case, the use of a shear correction factor, see Timoshenko [42], Cowper [22], Dong et al. [26], is required to overcome the approximation of a constant shear distribution over the cross-section. The de Saint-Venant principle [25] states that two statically equivalent loads produce equivalent stress and strain fields if they are evaluated at a sufficiently large distance from the loads. In other words, even though the fundamental assumption of classical models are satisfied, the loads and boundary condition may afflict the solution at a local level. Some examples can be seen in the works by Horgan and Simmonds [31], Tullini [43] and Lin et al. [32] regarding end-effects or by Bar-Yoseph and Avrashi [3] and by Bar-Yoseph and Ben-David [4] concerning the free edge singularity. Advanced models are therefore required to obtain reliable results in that portion of the structure.

The introduction of refined structural models allows the limitations introduced by the fundamental assumptions of the classical models to be overcome and the stress singularities due to local effects to be dealt with. Many refined one-dimensional models have been proposed over the last few decades, e.g. the use of warping functions, as proposed by Vlasov [44], allows the cross-section deformation to be included in the beam models. Cross-sectional warping plays an essential role in thin-walled structures as shown in the work by Friberg [29] and Ambrosini [1], where the warping function approach was used. Schardt [38] proposed a one-dimensional model for the thin-walled structures analysis where the displacement field was considered as an expansion around the mid-plane of the thin-walled cross-section. This approach, which is called the generalized beam theory (GBT), was used by Davies and Leach [23] and Davies et al. [24], and an extension to the analysis of composite material was proposed by Silvestre and Camotim [40]. The Variation Asymptotic Method, VAM, proposed by Berdichevsky [7], uses a characteristic cross-section parameter

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to build an asymptotic expansion of the solution. The application of this approach to one-dimensional structures can be seen in the work by Giavotto et al. [30]. Volovoi [45], Yu et al. [51] and Yu and Hodges [50] have extended this method to composite materials and beams with arbitrary cross-sections. Živković et al. [52] proposed a general beam formulation including the cross-sectional deformation. A similar approach has been used by Yoon et al. [49] and Yoon and Lee [48] that proposed the introduction of a cross-sectional refined kinematic able to deal with in- and out-of-plane warping.

Different computational models have been developed on the basis of the above mentioned structural theories. A numerical tool that is frequently used in structural analysis is the Finite Element Model, FEM, which allows the structural theory to be easily included in the computational code. FEM models usually include classical structural theory, Timoshenko in the beam case, but also try to include refined approaches. This is the case of the so called elements BEAM188 and BEAM189 that exploit a cross-sectional model to define *ad hoc* warping functions for each geometry, has proposed by Schulz and Filippou [39]. A unified formulation that is able to provide any order structural model has been proposed by Carrera [10]. This approach, called the Carrera Unified Formulation, CUF, has been used to derive both 1D [14] and 2D [11] theories. When one-dimensional models are considered, the CUF allows the cross-sectional displacement field to be described using a function expansion, and the accuracy of the results can be increased by just increasing the expansion order [18]. In most cases, refined models are required to describe local effects when high stress gradients are present. In other cases, the classical model assumptions are not satisfied in some regions of the structure. The use of a refined beam model over the whole domain therefore requires more computational costs than those necessary. The best solution would be to use refined models only in the region in which they are required and classical models elsewhere. The problem of *mixing* or *joining* different structural models is a well-known topic in literature. An exhaustive review of the state of the art can be found in the work by Wenzel [47]. When models with incompatible kinematics have to be joined, compatibility of the displacement should be imposed between the two domains. One of the possible approaches is to impose the compatibility condition at the boundary between the two models. Compatibility can be imposed using Lagrange multipliers, as shown in the work by Prager [34]; the same approach has been used in the frameworks of the CUF by Carrera et al. [15]. An extension of this approach was proposed by Ransom [36] in which a spline was used to couple two incompatible meshes. This approach is called three-field. An alternative approach, which includes Lagrange multipliers in the principle of virtual work, was introduced by Blanco et al. [9]. A second approach that can be used to join incompatible structural models involves creating an overlapping zone of the two domains. In this case, there is a smooth transition between the two kinematics. An example of this approach is the Arlequin method, proposed by Ben Dhia and Rateau [6], Ben Dhia [5]. In this case, compatibility in the shared area is imposed by using Lagrange multipliers. This approach has also been used in CUF frameworks, for example, in the work by Biscani et al. [8]. Finally, many approaches have been proposed that use a coarse mesh over the whole structure and only overlap a refined mesh in the area where complex phenomena are expected. Some examples are the s-FEM method proposed by Fish [28] and the model proposed by Park et al. [33]. The s-FEM method has been coupled with other refinement techniques, see Reddy and Robbins [37], such as the *p*- and *h*- refinements, where the element order and the element size are refined locally. These refinement approaches were presented by Babuska and Chandra [2], Szabo and Babuska [41] and Rachowicz et al. [35]. Except for the cases mentioned above, the use of refined models based on the CUF has been limited to constant kinematic models, that is, the kinematic assumptions were considered uniform over the whole structural domain. The benefits, in terms of accuracy and computational cost, that came from the use of refined one-dimensional models, with respect to classical approaches, have been pointed out in many published works [16,17] and will not be discussed here in detail. The present work has the aim to improve the efficiency of the well-know refined one-dimensional models introducing a node-dependent kinematic formulation able to adopt advanced kinematics only where required. This approach, in contrast with classical FE models, allows the accuracy to be improved using a refinement in the kinematic assumptions without any mesh refinement of the FEM model. This approach, in fact, allows different kinematics to be assumed at each node of a one-dimensional beam element. This permits: the accuracy of the model to be increased only in the part of the structure where this is required, a transition element to be created that is able to connect elements with different kinematics, classical beam models to be connected with shell and solid elements without any displacement discontinuities, as shown by Carrera and Zappino [20] that connect refined one-, two- and three-dimensional models, and global-to-local analysis to be performed with only one model. No *ad hoc* formulations have been introduced to achieve these results, the transition between different kinematics is in fact guaranteed by the shape functions used in the FE model. The use of the Carrera Unified Formulation, presented in [12], and the properties of the FEM allow the model to be derived in a compact form, which is called *fundamental nucleus*. This approach was presented in part by Carrera and Zappino [19], some preliminary results were shown in that work. The present paper extends the approach to different beam models, and provides an exhaustive and general theoretical formulation. Classical models and equivalent single layer models, see [14], based on a Taylor expansion, have been used where a low accuracy was required. Layer-wise models, see [21], have instead been used where a refined result was required. The theoretical model is presented in the first part of the paper. Several results have been proposed to assess the model. Finally an application to the thin-walled structures has been introduced. The results show the advantages introduced by the present model, in terms of computational cost and accuracy.

2. Node-dependent kinematic beam elements

The one-dimensional finite element introduced in the following sections allows different kinematics to be used at each beam node. If a 2-node element is considered, see Fig. 1, the kinematic assumptions used at node 1 can be different from those used at node 2. The capabilities of this element allow refined beam models to be used only at the beam nodes that require refined kinematics. Such features make the present element suitable for many applications: global to local analysis, transition elements, connection between different FEM models (1D to 2D or 2D to 3D). The use of a finite element formulation makes it possible to have a continuous transition of the kinematic assumption within the element without the need of any *ad hoc* formulation. The reference system shown in Fig. 2 is considered. The coordinate y stays on the axis of the beam while x and z lay on the cross-section Ω. The displacement vector can be written as:

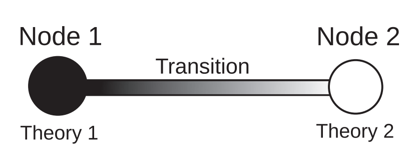

Fig. 1. A two-nodes beam element with a node-dependent kinematic.