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Modeling of composite plates with an arbitrary hole location using the variable separation method

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ABSTRACT

In this paper, a method to compute explicit solutions for laminated plate with arbitrary hole position is presented using a variable separation method. The displacement field is approximated as a sum of separated functions of the in-plane coordinates x, y, the transverse coordinate z and the coordinates X_T, Y_T of the hole position. As the parameterized problem involves the geometry, a mapping transformation is introduced to refer to a fixed reference configuration. This choice yields to an iterative process that consists of solving a 2D and three 1D problems successively at each iteration. In the thickness direction, a fourth-order expansion in each layer is considered. For the in-plane description, classical Finite Element Method is used. The functions of X_T and Y_T are discretized with linear interpolations. Mechanical tests with different numbers of layers are performed to show the accuracy of the method and its capacity to capture local effects and 3D state of the stress near the curved free-edge.

1. Introduction

Composite structures are widely used in the weight-sensitive industrial applications due to their excellent mechanical properties, especially their high specific stiffness and strength. For many cases of engineering application, the interlaminar stresses are very important for prediction of delamination which is one of the critical failure types of laminated systems. It is also well known that the presence of interlaminar stress components is particularly important in the regions close to free and loaded edges like bolted joints. Thus, a particular attention have to be paid to evaluate precisely their influence on local stress fields in each layer, particularly at the interface between layers.

The pioneering work on straight free-edge problems can be found in the review articles [1,2]. However, due to the complicated geometry and three-dimensional nature of the stresses, laminated composite plates with curved free-edges received relatively less attention. In the following, the references are limited to the works involving the modeling of laminated plate with a hole. Classically, 3D Finite Element (FE) approaches are used [3–7], but the computational cost can increase dramatically as a refined mesh is needed near the hole. Thus, the coupling with 2D approach far from the hole can be carried out [8], but the two regions have to be connected by dedicated formulations. To overcome that, 2D approaches have been developed and can be classified as follows:

- the Equivalent Single Layer Models (ESL): the Reissner-Mindlin (FSDT, [9,10]) and higher-order models (TSDT, [11]) have been addressed. But, only global quantities can be obtained, and the integration of the equilibrium equations is needed to estimate the transverse shear stresses with accuracy. Indeed, transverse shear and normal stress continuity conditions at the interfaces between layers are violated for all of them.
- the Layer-Wise Models (LW): It aims at overcoming the restriction of the ESL concerning the discontinuity of out-of-plane stresses at the interface between adjacent layers. These models are carried out in [11–13] with a high-order expansion of the displacements (at least three). Unfortunately, the number of unknowns depends on the number of layers.

It should be mentioned alternative approaches to overcome high computational cost. For that, the number of unknowns can be reduced by introducing the continuity conditions at the interface layers (on the transverse shear stresses). The so-called Zig-Zag models can be deduced as in [14]. Note also the use of hybrid stress elements based on a mixed form of Hellinger-Reissner principle in [15]. Both ESL and LW approaches are developed.

In this work, a promising alternative approach in the field of the reduced-order modeling based on the separation of variables [16] is performed to overcome these drawbacks and give the solution for any location of the hole. Note that the present study is focused on geometrical modification while preserving the topology of the geometry. One the one hand, it is based on the spatial coordinates separation proposed in [17] and also in [18], and the suitable order

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expansion given in [19–22] for the free-edge effects. On the other hand, a mapping transformation is used for the parameterization of the geometry to refer to a fixed configuration. It has been already carried out in the framework of the reduced basis in [23]. It has been also addressed in [24] with triangular domains for thermal problems and recently in [25] applied to non-straight lines for axisymmetric structures made of isotropic material. These two latter references are related to the proper generalized decomposition method. The present work aims at extending theses approaches to composite plates with a hole which can be considered as a serious challenging benchmark for the designer of layered structures. Thus, a representative test is addressed to assess the reliability and the effectiveness of the present method. The mapping transformation allows us to avoid complex integrations such as in [26]. Another way to solve this problem could be to use the fictitious domain with a indicator function for the hole as in [27]. Nevertheless, we are particularly interested in the transverse stresses near the hole. The representation of this indicator could influence the accuracy of the results in this zone of interest. Thus, this method is not chosen and we will ensure that the quality of the mesh remains good in this zone.

In our approach, the displacements are written under the form of a sum of products of bidimensional polynomials of (x, y), unidimensional polynomials of z and two unidimensional functions of the position of the hole X_T, Y_T. A piecewise fourth-order Lagrange polynomial of z is chosen. A linear interpolation is used for the functions of X_T and Y_T. As far as the variation with respect to the in-plane coordinates is concerned, a 2D eight-node quadrilateral FE is employed. The deduced non-linear problem implies the resolution of four linear problems alternatively. This process yields to a 2D and three 1D problems. Moreover, the explicit solution with respect to the hole position allows us to build a virtual chart in a straightforward manner avoiding the use of numerous expensive LW computations. This could be used to determine the influence of the hole position on the strength failure as in [28]. It could be also seen as a first step towards geometric optimization for laminates.

We now outline the remainder of this article. First, the mechanical formulation is recalled. The parameterization of the hole position requires a mapping transformation which is described. Then, the iterative algorithm to solve the non-linear problem introduced by the variables separation is detailed. The FE discretization is also described. Numerical evaluations are subsequently presented for one-layer and four-layer plate. The behavior of the method is first presented and illustrated. Then, it is assessed to capture local effects and 3D state of the stress, and in particular the transverse stresses near the curved edge, by comparing with reference model available in literature. Results of a LW model issued from Carrera's Unified Formulation [29] are used. Interesting features near the curved free edge can be emphasized even with involving very localized phenomenon.

2. Reference problem description: the governing equations

Let us consider a composite structure occupying the domain $\mathcal{V} = \Omega \times \Omega_z$ with $\Omega = [-a/2, a/2] \times [-b/2, b/2]$ and $\Omega_z = [-h/2, h/2]$ in a Cartesian coordinate (x, y, z). The plate is defined by an arbitrary region Ω in the (x, y) plane, located at the midplane for $z = 0$, and by a constant thickness h. See Fig. 1.

2.1. Constitutive relation

The plate can be made of NC perfectly bonded orthotropic layers. The constitutive equations for a layer k can be written as

$$\boldsymbol{\sigma}^{(k)}(z) = \mathbf{C}^{(k)}\boldsymbol{\varepsilon}(z) \quad \text{for} \quad z \in [z_k, z_{k+1}] \tag{1}$$

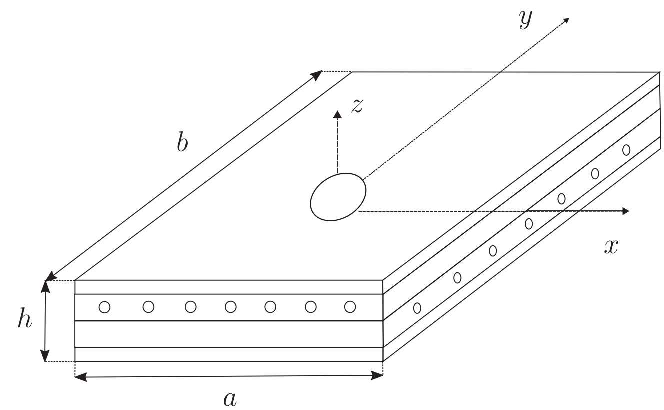

Fig. 1. The laminated plate and coordinate system.

where we denote the stress vector $\boldsymbol{\sigma}$, the strain vector $\boldsymbol{\varepsilon}$. z_k is the z-coordinate of the bottom surface of the layer k.

We have

$$\mathbf{C}^{(k)} = \begin{bmatrix} C_{11}^{(k)} & C_{12}^{(k)} & C_{13}^{(k)} & 0 & 0 & C_{16}^{(k)} \\ & C_{22}^{(k)} & C_{23}^{(k)} & 0 & 0 & C_{26}^{(k)} \\ & & C_{33}^{(k)} & 0 & 0 & C_{36}^{(k)} \\ & & & C_{44}^{(k)} & C_{45}^{(k)} & 0 \\ & sym & & & C_{55}^{(k)} & 0 \\ & & & & & C_{66}^{(k)} \end{bmatrix} \tag{2}$$

where $C_{ij}^{(k)}$ is the three-dimensional stiffness coefficients of the layer (k).

2.2. The weak form of the boundary value problem

The plate is submitted to a surface force density $\mathbf{t}$ defined over a subset Γ_N of the boundary and a body force density $\mathbf{b}$ defined in $\mathcal{V}$. We assume that a prescribed displacement $\mathbf{u} = \mathbf{u}_d$ is imposed on $\Gamma_D = \partial\mathcal{V} - \Gamma_N$.

Using the above matrix notations and for admissible displacement $\delta\mathbf{u} \in \delta U$, the variational principle is given by:

find $\mathbf{u} \in U$ such that:

$$-\int_{\mathcal{V}} \boldsymbol{\varepsilon}(\delta\mathbf{u})^T \boldsymbol{\sigma}\, d\mathcal{V} + \int_{\mathcal{V}} \delta\mathbf{u}^T\mathbf{b}\, d\mathcal{V} + \int_{\Gamma_N} \delta\mathbf{u}^T\mathbf{t}\, d\Gamma = 0 \qquad \forall\delta\mathbf{u} \in \delta U \tag{3}$$

where U is the space of admissible displacements, i.e. $U = \{\mathbf{u} \in (H^1(\mathcal{V}))^3 / \mathbf{u} = \mathbf{u}_d \text{ on } \Gamma_D\}$. We have also $\delta U = \{\mathbf{u} \in (H^1(\mathcal{V}))^3 / \mathbf{u} = 0 \text{ on } \Gamma_D\}$.

3. Application of the separated representation to plate with a hole

In this section, we introduce the application of the variables separation for composite plate analysis with an arbitrary hole location. The coordinates of the hole center are considered as parameters of the solution. The main idea consists in using a mapping to refer to a fixed configuration. It has been already used in [25] for parameterized geometry of axisymmetric structures made of isotropic material, and also in [30] for cylindrical shell applications. Moreover, the separated spatial representation [18] is considered with a suitable degree of interpolation of the transverse function for such structures as in [19].