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Dynamic analysis of a multi-span beam subjected to a moving force using the frequency domain spectral element method

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This study extends a frequency domain modified spectral element method (SEM) from single-span beams to multi-span beams subjected to moving point forces. Each span is represented by the Timoshenko beam model. The time history of the moving point force is transformed to the frequency domain as a series of quasi-static or stationary point forces acting on the beam simultaneously. The dynamic responses are obtained by superposing the individual dynamic responses excited by each quasi-static point force. The SEM based on the original one-element method provides the exact individual dynamic responses of all spans except for the span on which the quasi-static point force is located. Thus, the exact dynamic responses for this span are obtained based on the modified one-element method by adding some correction terms that are given in closed analytical forms. The method is highly accurate and computationally efficient, as verified by comparison with other techniques such as exact theory, modal analysis method, and finite element method.

1. Introduction

Beam structures subjected to moving loads may experience severe vibration that could result in structural failure. Examples of such structures include bridges and overhead cranes. Thus, accurate and efficient prediction of this vibration is very important for successful design and maintenance of such structures.

Numerous dynamic analysis methods have been proposed to predict the vibrations of beam structures subjected to moving loads. These include integral transforms [1–3], mode superposition or modal analysis [4–11], the transfer matrix [12–14], the generalized moving least square method [15], the Lagrange equation [16,17], the U-transformation and mode method [18], the modified beam vibration function [19], the Galerkin method [20], the finite element method (FEM) [21,22], the finite difference method [23], the dynamic stiffness method [24], and the frequency domain spectral element method [25–27].

The application of analytical methods is limited to very simple moving load problems, and computational methods, such as FEM have been widely used for most practical moving load problems. However, FEM generally requires a very fine structural discretization to obtain accurate solutions, particularly at high frequencies. This is necessary because the shape functions used in FEM are independent of the vibrating frequencies of a structure. In contrast to FEM, the frequency-domain spectral element method (SEM) provides extremely accurate solutions by representing a uniform beam structure member as a single finite element, regardless of its length [28,29]. This is accomplished by using the exact dynamic stiffness matrix as the stiffness matrix, which is formulated using the frequency-dependent shape functions derived from exact free wave solutions that satisfy the governing equations of motion.

Despite the extremely high accuracy and efficiency of SEM, very few studies have applied it to moving load problems [25–27]. Azizi et al. [25] seem to be the first to apply SEM to the dynamic analysis of continuous beams and bridges subjected to a moving force. A uniform beam structure was discretized into a large number of finite elements (more than two) and a moving point force was represented by effective nodal forces and moments acting on the two nodes of a finite beam element. This approach was commonly used in FEM-based dynamic analysis. However, it did not fully exploit the key advantages of SEM. Moreover accurate solutions could not be obtained because the effective nodal forces and moments are not mechanically equivalent to the original point force. The static Green's function was necessary for improving the solutions. Sarvestan et al. [26] later applied the same SEM technique to the vibration analysis of a cracked Bernoulli-Euler beam subjected to a moving load.

Recently, Song et al. [27] proposed a new SEM-based technique for beams subjected to a moving point force. They represented a

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moving point force in the frequency domain as a series of quasi-static point forces and superposed all the individual dynamic responses to obtain the exact dynamic responses. Accurate individual dynamic responses were obtained by representing a beam subjected to a quasi-static point force by two finite elements. Thus, a quasi-static point force was considered a nodal force acting on the joint node of two finite elements; this is called the two-element method.

A new SEM-based dynamic analysis technique, called the modified one-element method, was developed to avoid the structural discretization required for the two-element method [30]. This method was applied to uniform Timoshenko beams subjected to stationary dynamic forces. The modified one-element method provides accurate solutions by adding correction terms to the solutions obtained by the one-element method. The correction terms were given in analytical forms. The one-element method is an approach that is most commonly used in FEM, where a beam segment subjected to a point force is represented by a single finite element and the point force is represented by effective nodal forces and moments applied at the two nodes of the finite element. To the best of our knowledge, the modified one-element method has not been applied to the dynamic analysis of a beam subjected to a moving force.

Thus, the purpose of this study is to apply the modified one-element method to the dynamic analysis of multi-span beams subjected to a moving point force. Section 2 presents the problem statement, and Section 3 a frequency-domain representation of a moving point force as a series of quasi-static point forces. Section 4 presents the general procedure of dynamic analysis, and Section 5 various dynamic analysis methods for a multi-span beam subjected to a quasi-static point force. In Section 6, we discuss the derivation of frequency-domain dynamic responses using the modified one-element method. Section 7 presents numerical results that demonstrate the high accuracy and computational efficiency of the method in comparison with other techniques. The effects on the dynamic responses of single-span and multi-span beams of various parameters are investigated, including the boundary conditions, the moving speed and acceleration of a point force, and the time intervals of a series of moving point forces. Section 8 summarizes the results of the study.

2. Problem statement

A multi-span beam subjected to a moving point force is shown in Fig. 1. The transverse point force has constant magnitude P. The beam is made of elastic materials, and the vibration amplitudes are small. The beam consists of a total of Q spans, and its total length is L. The length of each span is denoted by l_q, where $q = 1, 2, 3, \ldots, Q$. In Fig. 1, two axial coordinates are introduced: the global coordinate X for the entire multi-span beam (with the origin at the left end of the first span) and the local coordinate x for a specific span (with the origin at the left end of the specific span). The current position of the moving point force with respect to the global coordinate is $S(t)$, whereas the distance from the left end of the first span ($X = 0$) to the right end of the q-th span is X_q. Using Timoshenko beam theory, the equations of motion for a uniform beam subjected to a transverse moving point force P can be written as

$$\boldsymbol{M}\frac{\partial^2 \boldsymbol{u}(X,t)}{\partial t^2} + \boldsymbol{K}\boldsymbol{u}(X,t) = \boldsymbol{f}(X,t) \quad (X_{q-1} \leqslant X \leqslant X_q) \tag{1}$$

where

$$\boldsymbol{u}(X,t) = \begin{Bmatrix} w(X,t) \\ \theta(X,t) \end{Bmatrix}, \quad \boldsymbol{f}(X,t) = \begin{Bmatrix} f(X,t) = P\delta(X - S(t)) \\ 0 \end{Bmatrix} \tag{2}$$

and

$$\boldsymbol{M} = \begin{bmatrix} \rho A & 0 \\ 0 & \rho I \end{bmatrix}, \quad \boldsymbol{K} = \begin{bmatrix} -\kappa GA\frac{\partial^2}{\partial X^2} & \kappa GA\frac{\partial}{\partial X} \\ -\kappa GA\frac{\partial}{\partial X} & \kappa GA - EI\frac{\partial^2}{\partial X^2} \end{bmatrix} \tag{3}$$

where $w(X, t)$ and $\theta(X, t)$ are the transverse displacement and the slope, respectively; ρ is mass density; A and I are cross-sectional area and the area moment of inertia, respectively; E and G are Young's modulus and shear modulus, respectively; and κ is the shear correction factor. In Eq. (2), $\delta(X)$ is the delta function [31].

3. Representation of a moving point force in the frequency domain

Using both the discrete Fourier transform theory (DFT) [32] and the discrete representation of a continuous function by delta functions [33], the moving point force $f(X, t)$ in Eq. (2) can be transformed into the frequency domain as follows [27]:

$$F(X,\omega) = \sum_{n=0}^{N-1} F_n \delta(X - S_n) \tag{4}$$

where

$$F_n = Pe^{-i\omega t_n} \quad (n = 0, 1, 2, \ldots, N-1) \tag{5}$$

and

$$S_n = S(t_n) \quad (n = 0, 1, 2, \ldots, N-1) \tag{6}$$

where $i = \sqrt{-1}$ is the imaginary unit, and $t_n = n\Delta t$. The time increment is defined by $\Delta t = T/N$, where T is the time window (or sampling time) and N is the total number of spectral components up to the Nyquist frequency to be considered in the FFT-based spectral analysis [32]. If the moving speed of the point force is constant, Eq. (6) can be written as $S_n = S(t_n) = vt_n$. If the moving speed is time-varying, Eq. (6) can be replaced with $S_n = S(t_n) = (1/2)at_n^2 + vt_n$, where a is the acceleration [27].

Fig. 2 shows the frequency-domain presentation of a moving point force as a series of quasi-static or stationary point forces defined by Eq. (4) and acting on the multi-span beam. The time window T is selected to be equal to T_A, where T_A denotes the traveling time of the point force from the left end ($X = 0$) to the right end ($X = L$) of the beam. The case $T = T_A$ can be readily extended to case $T \neq T_A$ by referring to Song et al. [27]. The position of the

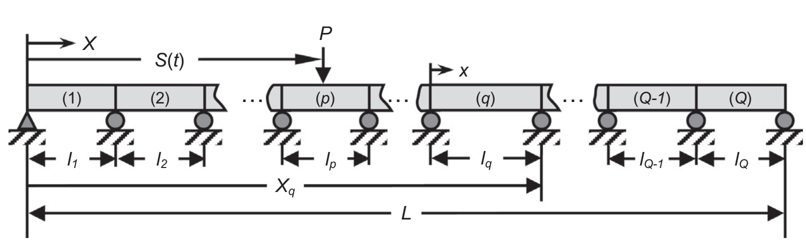

Fig. 1. Multi-span beam subjected to a moving point force P.