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A non-oscillatory time integration method for numerical simulation of stress wave propagations

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ABSTRACT

When the compressive loads are dominant in a composite structure, a tensile stress may be induced owing to the propagation of a stress wave and the interaction between an incident wave and a reflection wave, thus leading to the occurrence of cracks. Therefore, stress wave have a significant effect on the life of composite structures. In this study, a four sub-step explicit time integration scheme is proposed for solving stress wave propagation problems. This method builds on the fourth-order central difference method and a high-order derivative term to minimize the numerical oscillation. The proposed scheme possesses a first-order accuracy in the case of undamped and damped systems. Stability, accuracy, and dispersion of the proposed explicit direct time integration scheme are analyzed. Furthermore, the performance of this scheme is illustrated by the solution of a stress wave propagation and wave reflection in a one-dimensional impact problem and two-dimensional scalar wave propagation.

1. Introduction

The propagation of stress waves and wave reflection are important factors in a composite structure system. If a compressive wave propagates to a corner enclosed by two free surfaces at a certain angle, then net tensile stress may occur due to the interaction between the two reflective waves and thus cause the corner spalling [1]. The compressive loads are applied to the composite structure and then transverse cracks caused by the tensile stress may be initiated. Therefore, in the present study, we start the theoretical investigation of the stress wave in order to analysis the crack of structures under the compressive loads.

Stress wave propagation is investigated considering the inertia of the infinitesimal element of a structure under explosive or impact loading. The hyperbolic wave equations are solved by mathematical analysis. As mathematical analysis is performed under limiting conditions, numerical methods can be used for approximate solving of the wave equations. These numerical methods can be categorized into the following three groups [1]: characteristics method, finite difference method, and finite element method.

The characteristics method is usually applied to solve one-dimensional stress wave problems. However, in general multidimensional stress wave problems, as the characteristics and compatibility equations are fairly complicated, it is difficult to obtain a solution. The finite difference method has been widely applied to solve the problems of rectangular or simple shapes, where it is relatively easy to obtain a solution [2]. The feature of the finite element method is that it handles complicated geometries and boundary conditions. This has resulted in extensive finite element research and numerous studies about wave propagation have been performed using this method [2–9].

There are various general sources of errors in the finite element solution [2], such as the discretization [3,4,10–12], integration in space [4–6,13], constitutive relations [14–18], dynamic equilibrium equations [7,8,19–22], iteration [23–26], and round-off [27–29]. In the case of dynamic equilibrium equations, approximate finite element solutions are obtained by direct time integration schemes. Direct time integration is mainly used in stress wave propagation and dynamic structure problems. It is difficult to find accurate finite element solutions for stress wave propagation problems using the direct time integration method. The finite element solutions are not accurate owing to numerical errors occurring in temporal and spatial discretization, such as artificial period elongations, amplitude decays, numerical dispersions, dissipations, and oscillations [2–8].

In previous research, many direct time integration schemes were proposed to improve the accuracy of the solution. The time integral schemes were presented in the central difference method, Runge-Kutta method, Bathe method [7], Noh-Bathe method [8], Newmark method [19], Hulbert and Chung [21], and

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Tchamwa-Wielgosz method [22]. The Tchamwa-Wielgosz method is first-order accurate, and the central difference method, Bathe method, Noh-Bathe method, and Newmark method are second-order accurate. However, even when these methods are applied to solve the problems, the numerical oscillations and errors remain. In addition, the four sub-step Runge-Kutta scheme with fourth-order accuracy cause spurious oscillations of the solution [30].

Owing to numerical oscillations, the finite element solutions differ from exact solutions. Therefore, it is difficult to obtain accurate solutions and analyze the results by the finite element analysis of structures. For example, in Ref. [31], the numerical oscillations in contact pressure were presented using the finite element contact model. Furthermore, in Ref. [32], resulting from dynamic response and numerical effect, some numerical oscillations in the impact force appeared. Similarly, de Matos and Nowell [33] reported some difficulties using stress extrapolation, which was used to investigate the plasticity-induced fatigue crack closure according to the corner point singularities and the Poisson's ratio. In Ref. [34], the spurious oscillations may result in low accuracy when estimating a structure life by the Rainflow method. An important point for the fatigue limit state of structures is to reduce the errors from numerical oscillations. It is also important to reduce errors in wave propagation caused by spurious oscillations.

Despite numerous studies using the direct time integration method, there is no reliable integration method for the accurate non-oscillatory solution, even for wave propagation problems in elastic materials, which undermines the reliability of the numerical results in wave propagation solutions. However, in previous research, the finite element problems have been solved by using the filtering processing technique instead of an alternative direct time integration method. By filtering the specific time and space points, the errors from numerical oscillations can be reduced [5,35,36]. However, these approaches do not lend themselves to analyses requiring a solution for all times and over the complete solution domain [7].

In the present study, we propose the explicit time integration scheme to reduce the numerical oscillations in the propagation of stress waves. The procedure for the new explicit time integration scheme with first-order accuracy is developed and evaluated through stability and accuracy analyses, according to standard procedures described in details in Ref. [2]. Then we investigate the dispersion properties in one-dimensional and two-dimensional analyses. Finally, we provide the calculated response for the elastic bar wave problems and a two-dimensional wave problem using the proposed method, Bathe method, and central difference method.

2. Four sub-step explicit time integration scheme

2.1. Proposed explicit time integration scheme

The governing finite element equations in linear analysis are provided by Eq. (1):

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{R} \tag{1}$$

where $\mathbf{U}$, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$, and $\mathbf{R}$ are the nodal displacement vector, mass matrix, damping matrix, stiffness matrix and external nodal force vector, respectively, and the over-dot means differential with respect to time. With the displacements, velocities, and accelerations at time t, the solutions at time $t+\Delta t$ are calculated by the direct time integration scheme.

The basic approach of the proposed method is to use the fourth-order central difference method and a high-order derivative term, as inspired by the Kawamua-Kuwahara scheme [37]. The Kawamua-Kuwahara scheme is a numerical scheme that considers a fourth-order central difference equation and a fourth-order derivative term. In the proposed method, the time step Δt is divided into four sub-steps; then, unknown displacements, velocities, and accelerations are calculated.

In the proposed method, the first sub-step is as follows (see Eqs. (2)–(4)):

$$^{t+\Delta t/4}\mathbf{U} = {}^{t}\mathbf{U} + \frac{\Delta t}{4}{}^{t}\dot{\mathbf{U}} + \frac{1}{2}\left(\frac{\Delta t}{4}\right)^2 {}^{t}\ddot{\mathbf{U}} \tag{2}$$

$$^{t+\Delta t/4}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \frac{\Delta t}{4}{}^{t}\ddot{\mathbf{U}} \tag{3}$$

$$\mathbf{M}^{t+\Delta t/4}\ddot{\mathbf{U}} + \mathbf{C}^{t+\Delta t/4}\dot{\mathbf{U}} + \mathbf{K}^{t+\Delta t/4}\mathbf{U} = {}^{t+\Delta t/4}\hat{\mathbf{R}} \tag{4}$$

The second sub-step is as follows (see Eqs. (5)–(7)):

$$^{t+\Delta t/2}\mathbf{U} = {}^{t}\mathbf{U} + \frac{\Delta t}{2}{}^{t+\Delta t/4}\dot{\mathbf{U}} + \frac{1}{2}\left(\frac{\Delta t}{2}\right)^2 {}^{t+\Delta t/4}\ddot{\mathbf{U}} \tag{5}$$

$$^{t+\Delta t/2}\dot{\mathbf{U}} = {}^{t}\dot{\mathbf{U}} + \frac{\Delta t}{2}{}^{t+\Delta t/4}\ddot{\mathbf{U}} \tag{6}$$

$$\mathbf{M}^{t+\Delta t/2}\ddot{\mathbf{U}} + \mathbf{C}^{t+\Delta t/2}\dot{\mathbf{U}} + \mathbf{K}^{t+\Delta t/2}\mathbf{U} = {}^{t+\Delta t/2}\hat{\mathbf{R}} \tag{7}$$

The third sub-step is as follows (see Eqs. (8)–(10)):

$$^{t+3\Delta t/4}\mathbf{U} = {}^{t+\Delta t/4}\mathbf{U} + \frac{\Delta t}{2}{}^{t+\Delta t/2}\dot{\mathbf{U}} + \frac{1}{2}\left(\frac{\Delta t}{2}\right)^2 {}^{t+\Delta t/2}\ddot{\mathbf{U}} \tag{8}$$

$$^{t+3\Delta t/4}\dot{\mathbf{U}} = {}^{t+\Delta t/4}\dot{\mathbf{U}} + \frac{\Delta t}{2}{}^{t+\Delta t/2}\ddot{\mathbf{U}} \tag{9}$$

$$\mathbf{M}^{t+3\Delta t/4}\ddot{\mathbf{U}} + \mathbf{C}^{t+3\Delta t/4}\dot{\mathbf{U}} + \mathbf{K}^{t+3\Delta t/4}\mathbf{U} = {}^{t+3\Delta t/4}\hat{\mathbf{R}} \tag{10}$$

Finally, the fourth sub-step is as follows (see Eqs. (11)–(13)):

$$\begin{aligned} ^{t+\Delta t}\mathbf{U} = {} & 8^{t+3\Delta t/4}\mathbf{U} - 8^{t+\Delta t/4}\mathbf{U} + {}^{t}\mathbf{U} - 3\Delta t^{t+\Delta t/2}\dot{\mathbf{U}} \\ & + \alpha(\Delta t)^{4\,t+\Delta t/2}\mathbf{U}^{(4)} \end{aligned} \tag{11}$$

$$\begin{aligned} ^{t+\Delta t}\dot{\mathbf{U}} = {} & 8^{t+3\Delta t/4}\dot{\mathbf{U}} - 8^{t+\Delta t/4}\dot{\mathbf{U}} + {}^{t}\dot{\mathbf{U}} - 3\Delta t^{t+\Delta t/2}\ddot{\mathbf{U}} \\ & + \alpha(\Delta t)^{4\,t+\Delta t/2}\mathbf{U}^{(5)} \end{aligned} \tag{12}$$

$$\mathbf{M}^{t+\Delta t}\ddot{\mathbf{U}} + \mathbf{C}^{t+\Delta t}\dot{\mathbf{U}} + \mathbf{K}^{t+\Delta t}\mathbf{U} = {}^{t+\Delta t}\hat{\mathbf{R}} \tag{13}$$

where $\mathbf{U}^{(i)}$ is i-order differential and α is a parameter to be determined. The parameter α affects the stability limit and accuracy of the solution. $^{t+\Delta t/4}\hat{\mathbf{R}}$, $^{t+\Delta t/2}\hat{\mathbf{R}}$, and $^{t+3\Delta t/4}\hat{\mathbf{R}}$ are the load vectors corresponding to each sub-step and are given in Section 2.5. $^{t+\Delta t/2}\mathbf{U}^{(4)}$ and $^{t+\Delta t/2}\mathbf{U}^{(5)}$ are calculated using the following equations (see Eqs. (14) and (15)):

$$^{t+\Delta t/2}\mathbf{U}^{(4)} = \frac{^{t+3\Delta t/4}\dot{\mathbf{U}} - 3^{t+\Delta t/2}\dot{\mathbf{U}} + 3^{t+\Delta t/4}\dot{\mathbf{U}} - {}^{t}\dot{\mathbf{U}}}{(\Delta t)^3/32} \tag{14}$$

$$^{t+\Delta t/2}\mathbf{U}^{(5)} = \frac{^{t+3\Delta t/4}\ddot{\mathbf{U}} - 3^{t+\Delta t/2}\ddot{\mathbf{U}} + 3^{t+\Delta t/4}\ddot{\mathbf{U}} - {}^{t}\ddot{\mathbf{U}}}{(\Delta t)^3/32} \tag{15}$$

The first sub-step can be seen as the explicit Euler forward method. The second and third sub-steps are based on the second-order central difference method, while the fourth sub-step can be interpreted as using the fourth-order central difference method and a high-order derivative term. The fourth-order and fifth-order differentials are derived from the second-order central difference method and the Euler backward method.