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Theoretical prediction of the progressive buckling and energy absorption of the sinusoidal corrugated tube subjected to axial crushing

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ABSTRACT

A theoretical study is conducted to predict the progressive buckling and energy absorption of the sinusoidal corrugated tube subjected to axial crushing. Based on the super folding element theory, the stationary plastic hinge mechanism is proposed. The theoretical prediction of the progressive buckling and energy absorption is proposed by taking the eccentricity factor and amplitude factor into account. In the theoretical analysis, the idealized elastic-plastic material model is adopted and strain hardening effect is employed. Also, the new lower bound and upper bound of the solutions for the mean crushing force are obtained. The theoretical result can predict the crushing behavior of the circular tube which produces the axisymmetric ring mode under axial crushing. The mean crushing force is related to the eccentricity factor and the amplitude factor, but the total energy is independent of the eccentricity factor. The theoretical results are compared well with the numerical and experimental results of previous studies. The theoretical predicts of corrugated tube produce excellent characteristics in term of force-consistent and low crushing force and provide a reference to the research of the progressive buckling and energy absorption of corrugated tube subjected to axial crushing.

1. Introduction

Because thin-walled tube is light-weight and has high energy absorption, it have been widely used as energy absorbers to improve the structural crashworthiness and passenger safety of industrial products such as cars, trains, aircrafts and ships [1,2]. Superior energy absorption properties of thin-walled tube lay in the progressive and controllable plastic deformation modes [3]. Thus, investigation on crushing behavior of thin-walled tube is significant. The crushing behavior of thin-walled tube under axial crushing was investigated experimentally, theoretically and numerically by different researchers [4–10]. Early theoretical research about the plastic collapse of circular tube was based on the final deformation of the structures without considering the effect of loading procedure. One of the forehead studies on the axial crushing of thin-walled tube was performed by Alexander [4]. He presented static theoretical analyses for the axisymmetric behavior of circular tubes based on the experimental results. Abramowicz and Jones [5,6] showed that the deformation mode related to the material properties, shell geometry of circular tube. Although the traditional circular tube shows well mechanical property, research has shown that traditional circular tube under the axial loading produces the higher initial crushing force [11–12]. Andrews et al. [13] investigated the deformation modes and energy absorption properties of aluminum alloy circular tubes subjected to quasi-static axial crushing. They concluded that the deformation modes classified into the axisymmetric ring mode, the non-axisymmetric diamond mode and mixed mode (ring mode and diamond mode). The ring mode was the best mode of the circular tube as the energy absorption structure. The progressive model of circular tubes was developed by many subsequent researchers. Wierzbicki et al. [14] presented the basic concepts of the super folding element and the eccentricity factor m based on the theoretical model of Alexander. The eccentricity factor m is the ratio of the outward part to the total folding length. Singace et al. [15,16] studied the axisymmetric collapse of circular tubes with different geometrical ratios experimentally. They proposed the deformation fields based on three stationary plastic hinge mechanisms. As the fold developed, the deformation fields allowed both inward and outward radial deformation of the circular tube to develop. However, the circular tube hardly follows prescribed deformation mode which is available for conducting energy absorption properties of the structure to be maximum [17–20]. With the development of computer, the finite element methods have been used for the nonlinear dynamic analyses of structures. The implicit and explicit time integration schemes are used to

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analyze these problems. The classical method to solve the nonlinear dynamic problem is explicit time integration. The explicit time integration solution at a smaller time increments $t + \Delta t$ is based on the equilibrium conditions at a random time t [21]. The explicit time integration is using the central difference method. The time step size that can be employed without losing the stability of the algorithm must be smaller than, or equal to, the critical time step [22,23]. But a novel computational works differs from the classical solutions was proposed by Bathe et al. [21–23]. They investigated the nonlinear dynamic problem via the implicit time integration. The implicit time integration solution at a smaller time increment $t + \Delta t$ is based on the equilibrium conditions at the time increment $t + \Delta t$ [21]. The time step size of the explicit integration method is several orders of magnitude smaller than the step size of resolving the response accurately. They concluded that, in such cases, the effect of implicit integration method is better than that of the explicit integration method [21–23].

For improving the energy absorption properties and reducing the initial crushing force, the grooved tubes [24,25] and corrugated tubes [26,27] were used. The grooved configuration only affects the initial deformation moment, but the whole deformation moment is not been affected. Therefore, corrugated tube with a special corrugated surface is used widely. The purpose of the design is introducing the corrugated surface into the corrugated tube to produce axisymmetric mode, and the force-displacement relationship with desirable uniformity may be obtained. The corrugated surface is introduced along the axial direction causing large plastic bending moment at the same interval of the corrugated tube. Therefore, the large plastic hinge is formed at the same interval on the corrugated tube. Many researchers have been investigated the corrugated surface structures theoretically and experimentally [28,29]. With the spread of computer, the crushing behavior of corrugated surface structure can be numerically studied by software [30–33]. The energy absorption behavior of a metallic double-sine-wave beam under axial crushing was investigated by Jiang and Yang [20]. Buckling modes and energy absorption properties of sinusoidal patterns corrugated tubes under axial impact were researched by Liu et al. [34]. They concluded that the deformation modes could be classified into dynamic progressive buckling, dynamic plastic buckling, and transition buckling. Energy absorption behavior of metallic staggered double-sine-wave tubes under axial crushing was proposed by Hou et al. [35]. They verified that the double-sine-wave tubes could significantly reduce the initial crushing force and the magnitude of the fluctuant wave in load-displacement curve. The numerical analysis of aluminum foam filled corrugated single and double circular tubes under axial impact was investigated by Kılıçaslan [3]. However, the theoretical analysis of progressive buckling mode of corrugated tube subjected to axial crushing was presented by researchers rarely.

In the present study, by taking the eccentricity factor m and amplitude factor n into account, three stationary plastic hinge mechanisms and a novel theoretical model of progressive buckling mode are proposed. In addition, the theoretical expressions of mean crushing force and total energy are obtained, and the results are compared with the numerical and experimental results of the previous studies. This study aims to predict the progressive buckling mode and the energy absorption properties of corrugated tube. The results obtained from this study will provide a reference to the research of the progressive buckling mode of corrugated tube subjected to axial crushing.

2. Theoretical analysis

The progressive buckling mode of the sinusoidal corrugated tube subjected to axial quasi-static crushing is similar to that of the circular tube which produces the axisymmetric ring mode. Fig. 1 shows the progressive buckling of the sinusoidal corrugated tube subjected to axial quasi-static crushing. The sinusoidal corrugated tube is obtained by rotating a sine curve around a central line OZ. The sinusoidal expression is $y = a\ \sin(2\pi/\lambda)x$ with the wavelength λ, the amplitude a. The wavelength λ, the amplitude a and the arc length $2l$ of the sinusoidal corrugated tube is shown in Fig. 1(a). The progressive buckling processes of corrugated tube subjected to axial quasi-static crushing are shown in Fig. 1(b)–(e). The progressive buckling result in Fig. 1 is the simulation result of sinusoidal corrugated tube with $D = 50$ mm, $h = 1$ mm, $\lambda = 29.43$ mm, $a = 2$ mm. And the thickness h, the diameter D, the total length L is attached to the corrugated tube.

As mentioned above, the super folding element method was developed by Wierzbicki and Abramowicz [36,37] to predict the mean crushing force of thin-walled structures, and the theory was applied to the problem of progressive buckling of the thin-walled corrugated tube. The whole progressive buckling process contains a number of the complete folds. The fold generates from top to bottom of tube one by one, ultimately the folds are all generated. A complete fold is shown in Fig. 2. Within a complete fold, there are two parts. The first part (Part I) begins at point A and ends at point C (Fig. 2(a)–(c)); the second part (Part II) begins at point C and ends at point D (Fig. 2(c)–(e)). The points A, B, C, D, E, F and G represent the stationary plastic hinges. Because the thickness of the corrugated tube is thin, so the stresses σ_z, τ_{yz} and τ_{xz} along the thickness of the corrugated tube are very small, and the stress field in the thin-walled corrugated tube is approximated with a plane stress ($\sigma_x \neq 0$, $\sigma_\theta \neq 0$) for an axisymmetric progressive buckling [34]. The progressive buckling of corrugated tube can be modeled by stationary plastic hinge mechanism based on the idealized elastic-plastic material. For a complete fold (Part I and Part II) of corrugated tube, considering the energy equilibrium of the system, the total external work done by compression has to be dissipated by plastic deformation in bending and membrane. The mean crushing force is calculated from the global energy balance such that the sum of the bending energy and membrane energy of corrugated tube equal to the external work done. Therefore

$$P_m \cdot 2l\eta = E_b + E_m \tag{1}$$

where the half arc length l remains constant during the buckling. P_m is the mean crushing force, and η is the effective crush distance factor. In real structures, an actual fold cannot be completely flattened. The actual fold is less than $2l$ finally. E_b and E_m are the energy dissipated in bending deformation and the membrane deformation respectively.

During the first part, the factors, n and m, related to critical angle α_0^{I} and initial angle β_i^{I}, as shown in Fig. 3, are given by

$$\cos\alpha_0^{\mathrm{I}} = n + m, \quad \cos\beta_i^{\mathrm{I}} = 2n \tag{2}$$

where the amplitude $a = nl$, n is the amplitude factor and m is the eccentricity factor which defines the outward portion over the arc length $2l$ of a sinusoid cycle.

As the folding process goes on, the inclination of the first part element, $\overset{\frown}{AC}$, with respect to the horizontal is denoted by the angles, α^{I} and β^{I}. The geometry relationship between α^{I} and β^{I} is given by

$$\cos\beta^{\mathrm{I}} = \cos\alpha^{\mathrm{I}} - m + n \tag{3}$$

and the rate of angle are obtained by

$$\dot{\beta}^{\mathrm{I}} = \frac{\dot{\alpha}^{\mathrm{I}} \cdot \sin\alpha^{\mathrm{I}}}{\sqrt{1 - (\cos\alpha^{\mathrm{I}} - m + n)^2}} \tag{4}$$

During the second part, as shown in Fig. 4, the eccentricity factor is defined by $m' = 1 - m$ and is related to the critical angle β_0^{II}, and the amplitude factor n is related to the initial angle α_i^{II},