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Improving the accuracy of the dual Craig-Bampton method

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ABSTRACT

The objective of the work reported in this paper is to improve the well-known dual Craig-Bampton (DCB) method. The original transformation matrix of the DCB method is enhanced by considering the higher-order effect of residual substructural modes through residual flexibility. Using the new transformation matrix, original finite element models can be more accurately approximated in the reduced models. Herein, additional generalized coordinates are newly defined for considering the 2nd order residual flexibility. Additional coordinates related to the interface boundary can be eliminated by applying the concept of SEREP (the system equivalent reduction expansion process). The formulation of the improved DCB method is presented in detail, and its accuracy is investigated through numerical examples.

1. Introduction

In engineering practice, the degrees of freedom (DOFs) of numerical models have been continuously increased, along with the rapid increase in their complexity. When a complicated structure consisting with diverse components is designed through the cooperation of different engineers, it is very expensive to deal with its finite element models. This is because frequent design modifications affecting the whole and component models require repeated reanalysis. For these reasons, a number of model-reduction schemes have spotlighted its necessity, especially, in the structural dynamics community [1–36,40]. Among the proposed solutions, component mode synthesis (CMS) methods are considered very powerful solutions. With CMS methods, a large structural model is represented by an assemblage of small substructures; then is approximated using a reduced model constructed using only the dominant substructural modes. In CMS methods, it is important to select the proper dominant modes [2–4].

After pioneering work by Hurty [1] in the 1960s, numerous CMS methods have been introduced for various applications [5–34]. The CMS methods can be classified as fixed, free, and mixed-interface based methods, depending on how the interface is handled. The most successful fixed-interface based method is the Craig-Bampton method (CB method) [5] due to its simplicity, robustness, and accuracy. In contrast, the free-interface based methods [7–9] proposed earlier were not successful because those methods were not adequate for either accuracy or efficiency in spite of their important advantages. These included such as substructural independence and easy treatment of various interface conditions [9,20–22].

In 2004, Rixen [11] introduced a new free-interface based method as a dual counterpart of the CB method, namely, the dual Craig-Bampton (DCB) method. In the DCB method, Lagrange multipliers are employed along the interface for assembling substructures and thus an original assembled finite element (FE) model can be effectively reduced as a form of quasi-diagonal matrices, leading to computational efficiency. The most advantageous feature of the DCB method is that, when a substructure is changed, entire reduced matrices do not need be updated again because in the formulation, substructures are handled independently. This feature also makes it possible to assemble substructures even if their FE meshes do not match along the interface [16]. For all these reasons, the DCB method is an attractive solution for experimental-FE model correlation [17–19], as well as FE model updating and dynamic analysis considering various constraint conditions (contact, connection joint, damage, etc.) [20–22]. However, the DCB method still needs improvement in accuracy. In particular, the DCB method causes a weakening of the interface compatibility in reduced models, resulting in spurious modes with negative eigenvalues [11,14]. If the reduction basis chosen is not sufficient, such spurious modes may occur in lower modes, which is an obstacle to approximating the original FE model correctly.

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Recently, fixed-interface based CMS methods have been successfully improved considering the higher-order effect of the residual modes [10,24,25,29,30]. The motivation of this study is that the same principle can be adopted for improving free-interface based methods. In this study, we focus on improving the accuracy of the DCB method. We derive a new transformation matrix for the DCB method, improved by considering the 2nd order effect of residual substructural modes. One difficulty comes from the fact that the improved approximation of substructural dynamic behavior contains unknown eigenvalues. In the formulation, unknown eigenvalues are considered additional generalized coordinates. These are subsequently eliminated using the concept of the system equivalent reduction expansion process (SEREP) to reduce computational cost. Finally, improved solution-accuracy is obtained in the final reduced systems. Furthermore, the use of the present method avoids creation of spurious modes with negative eigenvalues in the lower modes.

In Section 2, we briefly review the original DCB method; and formulation of the improved DCB method is presented in Section 3. Section 4 describes the performance of the improved DCB method through various numerical examples and in Section 5, we explore the negative eigenvalues in lower modes for the original and improved DCB methods. Finally, conclusions are presented in Section 6.

2. Dual Craig-Bampton method

In this section, we briefly introduce the formulation of the dual Craig-Bampton (DCB) method, see Refs. [11,14,20,31] for detailed derivations.

In the DCB method, a structural FE model is assembled by N_s substructures as in Fig. 1a. The substructures are connected through a free interface boundary Γ (Fig. 1b). The compatibility between substructures is explicitly enforced using the following constraint equation

$$\sum_{i=1}^{N_s} \mathbf{b}^{(i)^T} \mathbf{u}_b^{(i)} = \mathbf{0}, \tag{1}$$

in which $\mathbf{u}_b^{(i)}$ is the interface displacement vector of the i-th substructure, and $\mathbf{b}^{(i)}$ is a signed Boolean matrix.

The linear dynamic equations for each substructure Ω_i can be individually expressed by

$$\mathbf{M}^{(i)}\ddot{\mathbf{u}}^{(i)} + \mathbf{K}^{(i)}\mathbf{u}^{(i)} + \mathbf{B}^{(i)}\boldsymbol{\mu} = \mathbf{f}^{(i)}, \quad i = 1, \cdots, N_s, \tag{2}$$

where $\mathbf{M}^{(i)}$ and $\mathbf{K}^{(i)}$ are the mass and stiffness matrices of the i-th substructure, $\mathbf{u}^{(i)}$ is the corresponding displacement vector, $\mathbf{f}^{(i)}$ is the external load vector applied to the substructure, and $\mathbf{B}^{(i)}\boldsymbol{\mu}$ is the interconnecting force between substructures with $\mathbf{B}^{(i)} = \begin{bmatrix} \mathbf{0} \\ \mathbf{b}^{(i)} \end{bmatrix}$ and the Lagrange multiplier vector $\boldsymbol{\mu}$. Note that $\ddot{()} = d^2()/dt^2$ with time variable t.

Assembling the linear dynamic equations for each substructure in Eq. (2) using the compatibility constraint equation in Eq. (1), the dynamic equilibrium equation of the original assembled FE model (see Fig. 1c) is constructed as

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}} \\ \boldsymbol{\mu} \end{bmatrix} + \begin{bmatrix} \mathbf{K} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{u} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \mathbf{f} \\ \mathbf{0} \end{bmatrix}, \tag{3}$$

with $\mathbf{M} = \begin{bmatrix} \mathbf{M}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{M}^{(N_s)} \end{bmatrix}$, $\mathbf{K} = \begin{bmatrix} \mathbf{K}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{K}^{(N_s)} \end{bmatrix}$, $\mathbf{u} = \begin{bmatrix} \mathbf{u}^{(1)} \\ \vdots \\ \mathbf{u}^{(N_s)} \end{bmatrix}$, $\mathbf{f} = \begin{bmatrix} \mathbf{f}^{(1)} \\ \vdots \\ \mathbf{f}^{(N_s)} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \mathbf{B}^{(1)} \\ \vdots \\ \mathbf{B}^{(N_s)} \end{bmatrix}$, where $\mathbf{M}$ and $\mathbf{K}$ are block-diagonal mass and stiffness matrices that consist of substructural mass and stiffness matrices ($\mathbf{M}^{(i)}$ and $\mathbf{K}^{(i)}$).

The global eigenvalue problem is defined for the original assembled FE model

$$\mathbf{K}_g(\boldsymbol{\varphi}_g)_i = (\lambda_g)_i \mathbf{M}_g(\boldsymbol{\varphi}_g)_i \quad \text{for } i = 1, \cdots, N_g, \tag{4}$$

with $\mathbf{K}_g = \begin{bmatrix} \mathbf{K} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{0} \end{bmatrix}$, $\mathbf{M}_g = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, in which $(\lambda_g)_i$ and $(\boldsymbol{\varphi}_g)_i$ are the global eigenvalue and eigenvector corresponding to the i-th global mode, respectively, and N_g is the number of DOFs in the original FE model. This number consists of interface and substructural DOFs ($N_g = N_b + \sum_{i=1}^{N_s} N_u^{(i)}$, where N_b is the number of interface DOFs and $N_u^{(i)}$ is the number of DOFs of the i-th substructure).

Because each substructure can be seen as being excited through interconnecting forces, the displacement of each substructure is assumed in the original DCB formulation, as

$$\mathbf{u}^{(i)} \approx -\mathbf{K}^{(i)^+}\mathbf{B}^{(i)}\boldsymbol{\mu} + \mathbf{R}^{(i)}\boldsymbol{\alpha}^{(i)} + \boldsymbol{\Theta}^{(i)}\mathbf{q}^{(i)}, \; i = 1, \cdots, N_s, \tag{5}$$

where $\mathbf{K}^{(i)^+}$ is the generalized inverse matrix of $\mathbf{K}^{(i)}$ (the flexibility matrix), $\mathbf{R}^{(i)}$ is the rigid body mode matrix, $\boldsymbol{\Theta}^{(i)}$ is the matrix that consists of free interface normal modes, and $\boldsymbol{\alpha}^{(i)}$ and $\mathbf{q}^{(i)}$ are the corresponding generalized coordinate vectors.

The rigid body and free interface normal modes of the i-th substructure are calculated by solving the following eigenvalue problems

$$\mathbf{K}^{(i)}(\boldsymbol{\varphi}^{(i)})_j = \lambda_j^{(i)}\mathbf{M}^{(i)}(\boldsymbol{\varphi}^{(i)})_j, \; j = 1, \cdots, N_u^{(i)}, \tag{6}$$

in which $\lambda_j^{(i)}$ and $(\boldsymbol{\varphi}^{(i)})_j$ are the j-th eigenvalue and the corresponding mode, respectively. Note that the mode vectors are scaled to satisfy the mass-orthonormality condition.

The free interface normal mode matrix $\boldsymbol{\Theta}^{(i)}$ in Eq. (5) consists of dominant and residual normal modes

$$\boldsymbol{\Theta}^{(i)} = \begin{bmatrix} \boldsymbol{\Theta}_d^{(i)} & \boldsymbol{\Theta}_r^{(i)} \end{bmatrix}, \tag{7}$$

in which $\boldsymbol{\Theta}_d^{(i)}$ and $\boldsymbol{\Theta}_r^{(i)}$ includes $N_d^{(i)}$ dominant free interface normal modes, and the remaining modes, respectively.

The displacement of the substructure can be approximated using only the dominant modes

$$\mathbf{u}^{(i)} \approx -\mathbf{K}^{(i)^+}\mathbf{B}^{(i)}\boldsymbol{\mu} + \mathbf{R}^{(i)}\boldsymbol{\alpha}^{(i)} + \boldsymbol{\Theta}_d^{(i)}\mathbf{q}_d^{(i)}, \tag{8}$$

where the term $-\mathbf{K}^{(i)^+}\mathbf{B}^{(i)}\boldsymbol{\mu}$ is the static displacement by interconnecting forces, and this term can be expressed using modal parameters

$$-\mathbf{K}^{(i)^+}\mathbf{B}^{(i)}\boldsymbol{\mu} = -\boldsymbol{\Theta}^{(i)}\boldsymbol{\Lambda}^{(i)^{-1}}\boldsymbol{\Theta}^{(i)^T}\mathbf{B}^{(i)}\boldsymbol{\mu} \quad \text{with}$$
$$\boldsymbol{\Lambda}^{(i)} = \text{diag}\left(\lambda_1^{(i)}, \lambda_2^{(i)}, \ldots \lambda_{N_u^{(i)}}^{(i)}\right), \tag{9}$$

where $\boldsymbol{\Lambda}^{(i)}$ is the substructural eigenvalue matrix.

Substituting Eq. (7) into Eq. (9), the static displacement can be divided into dominant and residual parts

$$-\mathbf{K}^{(i)^+}\mathbf{B}^{(i)}\boldsymbol{\mu} = -\boldsymbol{\Theta}_d^{(i)}\boldsymbol{\Lambda}_d^{(i)^{-1}}\boldsymbol{\Theta}_d^{(i)^T}\mathbf{B}^{(i)}\boldsymbol{\mu} - \boldsymbol{\Theta}_r^{(i)}\boldsymbol{\Lambda}_r^{(i)^{-1}}\boldsymbol{\Theta}_r^{(i)^T}\mathbf{B}^{(i)}\boldsymbol{\mu}, \tag{10}$$

with the corresponding substructural eigenvalue matrices $\boldsymbol{\Lambda}_d^{(i)}$ and $\boldsymbol{\Lambda}_r^{(i)}$ defined by

$$\boldsymbol{\Lambda}_d^{(i)} = \boldsymbol{\Theta}_d^{(i)^T}\mathbf{K}^{(i)}\boldsymbol{\Theta}_d^{(i)}, \; \boldsymbol{\Lambda}_r^{(i)} = \boldsymbol{\Theta}_r^{(i)^T}\mathbf{K}^{(i)}\boldsymbol{\Theta}_r^{(i)}. \tag{11}$$

Using Eq. (10) in Eq. (8), the following equation is obtained: