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Generalized warping and distortional analysis of curved beams with isogeometric methods

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ABSTRACT

Towards improving conventional beam elements in order to include distortional effects in their analysis, independent parameters have been taken into account in this study. Curved beam's behavior becomes more complex, even for dead loading, due to the coupling between axial force, bending moments and torque that curvature produces. Thus, the importance of simulating geometry exactly arises in order to approximate accurately the response of the curved beam. For this purpose, the isogeometric tools (b-splines and NURBS), either integrated in the Finite Element Method (FEM) or in a Boundary Element based Method (BEM) called Analog Equation Method (AEM), are employed in this contribution for the static analysis of horizontally curved beams of open or closed (box-shaped) cross sections. Responses of the stress resultants, stresses and displacements to static loading have been studied for various cross sections.

1. Introduction

Refined models either straight or curved with shell or solid elements are widely used in structures, such as for example the deck of a bridge with a thin-walled cross section, for stress or strain analysis. Over time, higher order beam theories have been developed to include nonuniform warping and distortional effects in beam elements which exhibit important advantages over more refined approaches [1]. The evaluation of the cross sectional properties, which are finally incorporated in the one-dimensional beam analysis, is associated with the accuracy of the model regarding the cross sectional behavior. Over the past decades, classical beam theories based on specific assumptions fail to describe accurately the structural behavior of beam elements, especially in more complex formulations such as in curved geometries. Among these theories, that of Saint-Venant (SV) still plays a crucial role due to the fact that the analysis reduces to the evaluation of warping and distortional functions over the cross sectional domain. However, this solution is exact for the uniform warping of a beam (warping/distortional deformations are not restrained). Towards improving SV theory, several researchers investigated the so-called SV's principle (stated in [2]) as well as the SV's end-effects in order to derive a more general formulation of beams' kinematics. In most of these studies, the solution is obtained as the sum of the SV'S solution and the residual displacements corresponding to the end-effects, as it will be later explained.

In the majority of past research works, thin-walled cross sections have been studied due to their low self-weight comparing to solid ones and, thus, their use in practice. These cross sections are more susceptible to torsional and distortional effects. Vlasov (1961) in [3] presented the Thin Tube Theory (TTT) and treated different cross section types as special cases of this general theory. Kollbrunner and Basler (1969) in [4] and Heilig (1971) in [5] were later reformulated TTT for multi-cell boxes with arbitrary cross sections. Kristek (1970) in [6] obtained analytical solution for simple practical cases and separated the analysis of transverse distortion from that of torsion with longitudinal warping employing the superposition principle. Wright, Abdel-Samad and Robinson (1968) in [7] studied the distortional warping response of single-cell box girders with longitudinally and transversely stiffened plates employing the beam on elastic foundation (BEF) analogy. Steinle (1970) in [8] tackled the torsional distortion problem and introduced distortional stress resultants in the analysis. Kollbrunner and Hajdin (1975) in [9] dealt with the extension of the beam theory of prismatic folded structures to include the deformation of the cross section for open and closed cross sections including warping. Other research efforts later expanded TTT considering only box-shaped cross sections (single- or multi-cell) and, thus, being not general [10–15]. Schardt (1989, 1994) in [16,17]

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developed an advanced formulation known as Generalized Beam Theory (GBT) which is a generalization of the classical Vlasov beam theory in order to incorporate flexural and torsional distortion. A distinguishing feature of GBT stems from the general character of its cross sectional analysis which enables the determination of cross-section deformation modes as well as their categorization to global, distortional or local ones. Discretization within the frame of GBT cross sectional analyses depends on the topology of nodes (dependent, independent or intermediate nodes for branched or unbranched sections) and cross sectional shapes (open- or closed-shaped). Further developments of GBT avoid some of its cumbersome procedures through eigenvalue cross sectional analysis [18–23]. The classification of the cross sections with respect to their geometry is not needed with this approach and the most important shape modes are obtained (compared to those derived by GBT). These approaches are employed nowadays by several researchers. Camotim, Silvestre and co-researchers expanded the method to cover a variety of cross sections, orthotropic materials, as well as geometrically nonlinear and inelastic problems [24–26]. Towards solving the problem for arbitrarily shaped homogeneous or composite cross sections, El Fatmi and Ghazouani (2011) in [27] presented a higher order composite beam theory (denoted HOCBT) that starts from the exact expression of SV's solution and introduces in- and out-of-plane independent warping parameters for symmetric orthotropic cross sections with the ability to extended it for arbitrary ones. However, in-plane warpings are only due to the flexural and axial deformation modes and, thus, it could be stated that this research effort studies Poisson ratio effects rather than distortional ones. Ferradi and Cespedes (2014) in [28] presented the formulation of a 3D beam element solving an eigenvalue problem for the distortional behavior of the cross section (in-plane problem) and computing warping functions separately by using an iterative equilibrium scheme. Genoese, Genoese, Bilotta and Garcea (2014) in [29] developed a beam model with arbitrary cross section taking into account warping and distortion with their evaluation being based on the solution of the 3D elasticity problem for bodies loaded only on the terminal bases and a semi-analytic finite element formulation. Finally, Dikaros and Sapountzakis (2016) in [30] presented a general boundary element formulation for the analysis of composite beams of arbitrary cross section taking into account the influence of generalized cross sectional warping and distortion due to both flexure and torsion. In this proposal, distortional and warping functions are evaluated by the same eigenvalue problem and in order of importance.

Regarding horizontally curved beams subjected to vertical or radial loads, they inherently exhibit a more complex behavior comparing to straight formulations due to the fact that the effects of primary and secondary torsion are always coupled to those of bending and cross section distortion either for centered or eccentric loads. Dabrowski (1968) in [31] elaborated Vlasov's theory and introduced distortional behavior of box girders with a symmetric cross section. His model introduces the distortion angle as the single degree of freedom which measures the magnitude of the cross-sectional distortion. Bazant and Nimeiri (1974) in [32] proposed the skew-ended finite element in order to implement the theory of non-uniform torsion for straight or curved thin-walled cross sections. Oleinik and Heins (1975) in [33], and Heins and Oleinik (1976) in [34] employing Vlasov's and Dabrowski's theories studied the structural behavior of curved box girders. In-plane deformations were approximated using a differential equation which was solved employing the finite difference method. In addition to these research efforts, Sakai and Nagai (1981) in [35], and Nakai and Murayama (1981) in [36] presented several results on the design procedures of the intermediate diaphragms for curved girders and noted that these play a very important role in moderating distortional warping of girders. Meanwhile, Martin and Heins (1978) in [37] expanded Dabrowski's equation, which predicts the cross-sectional deformations, so that the angular deformations induced at given points along an I-girder curved bridge can be calculated. Zhang and Lyons (1984) in [38,39] employed Dabrowski's theory combined with Finite element method to develop a multi-cell box element for the analysis of curved bridges. Nakai and Yoo (1988) in [40] presented an extended study on the analysis and design of curved steel bridges. Yabuki and Arizumi (1989) in [41] employing BEF analogy for distortion proposed spacing provisions which can be utilized for steel-plated intermediate diaphragms. Razaqpur and Li (1994) in [42] extended their previous theory to curved thin-walled box beams. Petrov and Geradin (1998) in [43] employing the same concept with El Fatmi and Ghazouani [27] for straight beams formulated a theory for curved and pre-twisted beams of arbitrary homogeneous cross sections, covering geometrically nonlinear range as well. Kim and Kim (2002) in [44] developed a theory for thin-walled curved beams of rectangular cross section by extending the theory developed earlier for straight beams taking into account warping and distortional deformations. Park, Choi and Kang (2005) in [45] expanded their previous work [13], which was limited to straight box girder bridges, to curved formulations. They developed a curved box beam element which was employed in order to develop design charts for adequate spacing of the intermediate diaphragms of curved bridges. Flexural and torsional displacement functions have been based on those proposed for doubly symmetric cross section by Kang and Yoo (1994) in [46] while distortional functions have been derived for a mono-symmetric cross section as in [12]. Despite the practical interest of their study, their proposal cannot accommodate elastic constraints and due to other assumptions made lacks of generality. In other research efforts, the vibration problems of thin-walled curved box girder bridges due to moving loads have been investigated. The curved box girder bridges have been numerically modelled using finite elements which take into account the torsional warping, distortion and distortional warping [47–49]. Other recent research efforts as the following ones mainly constitute design guides with new formulae for specific practical cases rather than a generalized theory for the analysis of curved beams. Particularly, in the study of Zhang, Hou, Li and Wang (2015) [50], a curved girder is simplified to straight one by using the M/r method and calculation formulae for determining the required diaphragm spacing are obtained by regression analyses. Yoo, Kang and Kim (2015) in [51] applied the concept of the BEF analogy for the analysis of distortional stresses of horizontally curved box-girders. The proposed procedure is capable of handling simple or continuous single cell box girders (or separated multi-cell box girders) with rigid or deformable interior diaphragms or cross-frames. Towards establishing a more general theory, Arici and Granata (2016) in [52] employed the Hamiltonian Structural Analysis Method for the analysis of straight and curved thin-walled structures on elastic foundation extending the so-called GBT. To the authors' knowledge, there are no research efforts that introduce a unified distortional and warping eigenvalue analysis of arbitrarily shaped cross sections to the analysis of curved beams.

In modern regulations and design specifications, the importance of torsional and distortional effects in stress or strain analysis of structural members is recognized. Particularly, in sub-sections 6.2.7.1 and 6.2.7.2 of EN 1993-2, Eurocode 3: Design of steel structures – Part 2: Steel bridges, regarding torsion, the designer is obliged to keep the distortional stresses under a specific limiting value or follow some general design rules in case of neglecting distortion. These are presented in clauses (1)–(9) of section 6.2.7, regarding torsion, of EN 1993-1-1, Eurocode 3: Design of steel structures – Part 1-1: General rules and rules for buildings. Nevertheless, no guidelines and specific modelling methodologies