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Finite element modeling of wear using the dissipated energy method coupled with a dual mortar contact formulation

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ABSTRACT

A finite element formulation for the simulation of two-dimensional frictional contact problems undergoing wear effects is presented. The approach considers multibody contact and accounts for finite configuration changes due to both deformation and wear. The contact discretization is based on the Mortar method while the enforcement of contact constraints is fulfilled with Lagrange multipliers defined in a linear or a quadratic dual-basis. The effects of wear are predicted with the Dissipated energy method using an internal state variable approach that increases the distance between worn bodies by an additional gap. The frictional conditions with wear are enforced using nonlinear complementarity functions, which are solved using a semi-smooth Newton method. A strategy for modeling the contact surface evolution and alleviate the deformation induced element distortion is also presented. The consistent linearization of all quantities yields a robust and efficient algorithm. Numerical examples demonstrate the performance of the proposed framework.

1. Introduction

Wear is a complex physical phenomenon that is characterized by the removal of material from the surface of a body due to the frictional contact interaction with another surface. The wear evolution is typically slow and essentially depends on the materials and surface properties of the contact pair together with the contact conditions. Several attempts have been made to both understand and model the evolution of wear under different scenarios. A comprehensive literature review of wear models and predictive equations was presented in [1].

In order to quantify wear, the Archard phenomenological model [2] is commonly applied. This model establishes a relation between the wear volume and the work of the normal force through the sliding distance. It employs the input variables from the contact setting such as the prescribed normal load, the displacement amplitude, the friction coefficient and hardness of the softer material to calculate the wear loss [3]. An alternative model for the evolution of wear is the so-called Dissipated energy method [4,5]. This model creates a relation between the wear volume and the accumulated dissipated energy through an independent coefficient: the energy wear coefficient. Recent studies have shown that the Dissipated energy method can obtain rather consistent results in problems involving small amplitude of the relative sliding motion [6,7] and in problems with unidirectional sliding [8,9]. Therefore, this method was adopted in this work.

The evolution of wear can strongly influence the contact problem. From one side, the material loss in the contact zone can promote significant shape changes and lead to the redistribution of contact tractions. This, on the other hand, can considerably affect the evolution of wear. Due to the coupling effects involved, the solution of this class of problems is a challenging task and the subject of current research. In the literature, two classes of contact algorithms including wear in finite element analysis can be found. In the first class, the contact problem is solved and the evolution of wear is computed in a post processing step [10]. Then, the accumulated wear is added to the normal gap [11,12] which changes the contact conditions by increasing the distance between the two bodies. This approach neglects the shape changes of the contact surface on the numerical solution and is mainly applicable for problems that create small amounts of wear [13–17]. In the second class, the contact surface evolution due to the accumulated wear is modeled and the coupling between deformation and wear is included in the numerical treatment. Several strategies have been proposed in this case. One approach consists in performing the finite element analysis of the contact problem and then the evaluation of the wear depth [7,18,19]. The geometry of the

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contact surface is updated according to the computed wear depth and the nodes within the so-called wear-box are repositioned in order to mitigate the distortion of degenerated finite elements. A different approach consists in employing and adaptive remeshing algorithm to overcome the distortion of elements [20]. In [21] the evolution of the contact surface is modeled by introducing a time-dependent material configuration that corresponds to the undeformed body of the shape changed due to wear.

A frictional contact formulation for the simulation of progressive wear effects in finite deformation problems is presented. The necessary contact surface discretization is based on the Mortar method to avoid the well know problems of node-to-segment contact formulations and to obtain optimal convergence rates from the finite element solution [22]. We follow an approach developed by [23] that is based on a continuous normal field of the discretized contact surface for definition of the mortar segments. The enforcement of contact constraints is realized with dual Lagrange multipliers, which can be locally eliminated from the global system of equations by static condensation. We consider both linear and quadratic dual basis for the Lagrange multipliers coupled with linear and quadratic finite elements. The objective is to assess the improvement of the quadratic dual basis for problems where the contact tractions are very sensitive to the curvature of contacting surfaces discretized with non-matching meshes and wear problems. In these cases, the error computing the gap with the Mortar method can have a relevant impact on the global error of the finite element solution [24]. The modeling of wear is undertaken with an internal state variable whose evolution is based on the Dissipated energy method. This variable is directly inserted into the contact constraints enforcing a modified non-penetration condition accounting for wear effects. The contact, friction and wear inequality constraints are reformulated in a set of so-called complementarity functions. For problems where the rate of wear depth is small compared to the deformation related to the contact forces, it might be possible to use the formulation from Refs. [25,26] to calculate the complementary functions. Nevertheless, for problems where the rate of wear is important, such as steady-state wear processes with large increments, the influence of the rate of wear depth must be considered on the solution within one time step to avoid convergence failure. Therefore, in this proposed framework, the effect of the wear is considered in the evaluation of the complementarity functions as well. The combination of the complementarity functions with the equilibrium equations, which include nonlinear constitutive material behavior at finite strains [27–31], leads to a system of nonlinear and non-differentiable equations that can be solved in terms of a semi-smooth Newton method. This modeling approach is mainly suitable for problems leading to a small amount of wear and encompasses the so-called Lagrangean step. To our knowledge, this is the first implementation of a two dimensional mortar contact formulation with wear at finite strains in the context of dual Lagrange multipliers with consistent linearization. Nevertheless, the modeling of the progressive evolution of the contact surface can be critical whenever shape changes due to deformation and wear are finite. Therefore, in the present work, a strategy for updating the geometry of the contact surface and for reallocating the surrounding nodes is proposed. In this scenario, the previously mentioned Lagrangean step is followed by a shape evolution step within a single time interval. This approach allows for significant wear loss.

Throughout this contribution, we focus on the extension of the Mortar formulation towards wear and refer to [32], and references therein, for the finite inelastic strain formulation employed in this work. In Section 2, the description of the two body frictional contact problem with wear in the context of finite deformations is undertaken. The governing equations of the Dissipated energy method are presented in Section 3. The boundary value problem is converted into a weak formulation in Section 4. The spatial discretization of the contact virtual work, the wear energy and the nonlinear contact constraints based on dual Lagrange multipliers are described in Section 5. Then, in Section 6 the solution procedure employed, based on the so-called primal-dual active set strategy with complementary functions is presented. The material removal modeling strategy adopted is addressed in Section 7. Numerical examples are provided in Section 8 to highlight the accuracy and robustness of this framework. Final remarks are given in Section 9.

2. Problem definition

A two body finite deformation frictional contact problem with wear is shown in Fig. 1. The bodies are represented by the open sets Ω_0^s and Ω_0^m, $\{\Omega_0^s \cup \Omega_0^m = \Omega_0 : \ \Omega_0 \subset \mathbb{R}^2\}$ in the reference configuration and the superscripts s and m represent the common nomenclature employed in contact mechanics of a *Slave* and a *Master* body. The boundaries of subset Ω_0 are divided in a contact zone, $\Gamma_c = \{\Gamma_c^s \cup \Gamma_c^m\}$, a Neumann boundary, Γ_N and a Dirichlet boundary Γ_D.

The symbol φ represents the mapping between the reference configuration $\mathbf{X}$ (at time 0) and the current configuration $\mathbf{x}$ (at time t) while $\hat{\mathbf{x}}^m$ is the closest projection of $\mathbf{x}^s$ onto the master surface, γ_c^m, in the current configuration (see Fig. 1). The same relation applies to $\widehat{\mathbf{X}}^m$ and $\mathbf{X}^s$ in the reference configuration. The Boundary Value Problem (BVP) given in terms of the vector of nodal displacements, $\mathbf{u}$ and the Cauchy stress tensor, $\boldsymbol{\sigma}$, as follows,

$$\begin{aligned} &\operatorname{div}\left(\boldsymbol{\sigma}^i(\mathbf{u})\right) + \hat{\mathbf{b}}^i = \mathbf{0} && \text{in } \Omega_t^i, \\ &\boldsymbol{\sigma}^i(\mathbf{u})\boldsymbol{\eta}^i = \hat{\mathbf{t}}^i && \text{on } \gamma_N^i \\ &\mathbf{u}^i = \varphi\left(\mathbf{X}^i, t\right) - \mathbf{X}^i = \hat{\mathbf{u}}^i && \text{on } \gamma_D^i, \quad i = s, m. \end{aligned} \tag{1}$$

where $\hat{\mathbf{b}}^i$ is the body force experienced by the solids in the current configuration, Ω_t^i. The spatial counterparts of the three boundaries are denoted by γ_c, γ_N and γ_D. The current outward unit normal vector on the Neumann boundary, γ_N^i, is denoted by $\boldsymbol{\eta}^i$. Prescribed displacements on the Dirichlet boundary, γ_D^i, are represented by $\hat{\mathbf{u}}^i$ and prescribed tractions on the Neumann boundary by $\hat{\mathbf{t}}^i$. The index i emphasizes that the BVP conditions must be attained for all bodies involved. Quasi-static equilibrium is assumed. For the solution of the BVP, a hyperelastic-based multiplicative framework is employed together with an algorithmic approach for modeling the behavior of nonlinear dissipative materials. This procedure is extensively discussed in [32].

In order to enforce non-penetration between the contact surfaces, the Kuhn-Karush-Tucker (KKT) conditions must be fulfilled. However, since the problem also includes the evaluation of material loss during the contact interaction, a new positive variable must be introduced: the wear depth w. This variable will be treated as an internal state variable that represents an additional gap, which is defined along the current outward normal vector on the slave surface (see Fig. 1). This approach was firstly introduced in [33]. The KKT conditions in the normal direction are expressed by the following set of constraints,

$$g(\mathbf{X}^s, t) + w(\mathbf{X}^s, t) \geqslant 0, \tag{2}$$

$$p_\eta \leq 0, \tag{3}$$

$$p_\eta[g(\mathbf{X}^s, t) + w(\mathbf{X}^s, t)] := 0. \tag{4}$$