Computers and Structures 191 (2017) 80–99

Contents lists available at ScienceDirect

Computers and Structures

journal homepage: www.elsevier.com/locate/compstruc

Chaotic dynamic buckling of rectangular spherical shells under harmonic lateral load

J. Awrejcewicz [a,b,*], A.V. Krysko [c,d], M.V. Zhigalov [e], V.A. Krysko [e]

[a] Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, 1/15 Stefanowski St., 90-924 Lodz, Poland
[b] Department of Vehicles, Warsaw University of Technology, 84 Narbutta Str., 02-524 Warsaw, Poland
[c] Cybernetic Institute, National Research Tomsk Polytechnic University, 30 Lenin Avenue, 634050 Tomsk, Russian Federation
[d] Department of Applied Mathematics and Systems Analysis, Saratov State Technical University, 77 Politehnicheskaya Str., 410054 Saratov, Russian Federation
[e] Department of Mathematics and Modeling, Saratov State Technical University, 77 Politehnicheskaya Str., 410054 Saratov, Russian Federation

ARTICLE INFO

Article history:
Received 24 March 2016
Accepted 16 June 2017

Keywords:
Chaos
Vibrations
Non-linearity
Shells
Fnite difference method
Faedo-Galerkin method

ABSTRACT

Dynamic bucking criteria for spherical shells of a rectangular form under sinusoidal lateral load are proposed and developed taking into consideration geometric and physical non-linearity. A mathematical model of thin shallow shells is constructed on the basis of the Kirchoff-Love hypothesis and the von Kármán geometric non-linearity, whereas the physical non-linearity follows the Ilyushin theory of plastic deformations. Reliability of the results is proved by comparing them with the results obtained by means of higher-order approximations of the Faedo-Galerkin method. Three scenarios (Feigenbaum, Ruelle-Takens-Newhouse and Pomeau-Manneville) are detected while transiting from regular to quasi-periodic/chaotic vibrations.

1. Introduction

Great effort has been carried out in aircraft, ship building, marine structures, aeronautic industries as well as robotics, microdevices fabrication and bioengineering to improve high levels of strength-to-weight and stiffness-to-weight relations while reducing the live cycle costs of the advanced materials either of conventional or composite structures simultaneously keeping their required dynamical stability performance and structural integrity.

In particular, engineers working in the aeronautical and aerospace industries are awaiting for reliable and validated design recommendations and criteria dedicated to further weight savings by taking into account the possibilities of safe operation of the structures in either buckling or postbuckling regimes.

These challenging demands require investigation of non-linear complexity of design and geometry, loading ways and conditions, mixed boundary conditions, initial imperfections, and in many cases also thermal and electromagnetic fields interaction. The mentioned technologically motivated requirements imply updating of the mathematical modeling of the structures/structural members to achieve validated and reliable prediction of the buckling and post buckling states as well as their possible damage and collapse.

Dynamic buckling of structural members has a long history in mechanics and include seminal works of Budiansky, Hutchinson and Elishakoff [1–3]. Numerical study of the buckling and initial post-buckling behaviour of clamped shallow spherical shells subject to axisymmetric ring load has been carried out by Akkas and Bauld [4]. Ball and Burt [5] proposed a novel criterion for dynamic buckling under the nearly axisymmetric load of the shallow spherical shells. The authors determined critical buckling pressures for a large range of shell sizes.

More recently the state-of-the art of vibrations of plates and shells including stability problems with emphasis on novel nonlinear phenomena exhibited by the mentioned structural members has been addressed by Amabili [6].

In the traditional approach, dynamic buckling phenomena concern fastly increasing in-surface compressive loads or time dependent in-surface displacement of the boundaries. In the beginning of the buckling oriented research, the problems have been usually modeled by linear/non-linear second order ODEs with constant or time-dependent coefficient, i.e. by one-degree-of-freedom

* Corresponding author at: Department of Automation, Biomechanics and Mechatronics, Lodz University of Technology, 1/15 Stefanowski St., 90-924 Lodz, Poland.

E-mail addresses: awrejcew@p.lodz.pl (J. Awrejcewicz), anton.krysko@gmail.com, krysko@tpu.ru (A.V. Krysko), zhigalovm@ya.ru (M.V. Zhigalov), tak@san.ru (V.A. Krysko).

http://dx.doi.org/10.1016/j.compstruc.2017.06.011
0045-7949/

systems and typically by taking into account either step or impulsive load (see [7] for more references).

Non-linear dynamics of isotropic shallow spherical shells has been studied by Budiansky and Roth [8] (the Galerkin method has been used) and Simitses [9], where the Ritz-Galerkin approach has been employed. Both finite difference method (FDM) and finite element method (FEM) have been applied based on the displacement buckling criterion by Kao [10], Saigal et al. [11], and Yang and Liaw [12]. Liaw and Yang [13] studied symmetric/asymmetric dynamic buckling of laminated thin shells by taking into account orthotropic and anisotropic material properties, axisymmetric and asymmetric imperfections and Rayleigh viscous damping. In addition, Huyan and Simitses [14] studied the problems of dynamic buckling of geometrically imperfect cylindrical shells subject to both axial compression and bending moment by FEM with an emphasis given to estimation of the dynamic critical loads.

The non-linear axisymmetric dynamics of clamped laminated angle-ply composite spherical caps subject to impact-type loads of infinite duration has been studied by Ganapathi et al. [15], and the obtained solution has been validated through the analytical/3D FEM analysis. Wei et al. [16] have investigated analytically and numerically the dynamic buckling of thin isotropic thermo-viscoplastic cylindrical shells compressed with an uniform axial velocity. The thin-walled carbon fiber reinforced shell structures under axial compression have been studied using FEM by Bisagni [17]. It has been shown that in the case of short time duration, the dynamic buckling loads were larger than the static ones, whereas increase of the load duration yielded decrease of the dynamic buckling load quickly achieving significantly smaller values than those regarding the static loads.

In spite of the purely numerical approaches aimed on investigation of dynamic buckling of shells, there exist also more theoretically oriented approaches focused on employing Hamiltonian formulation. For example, Steele and Kim [18] proposed a modified mixed variational principle and the state-vector equation yielding the so called Hamiltonian canonical equation with spatial/independent variables. This idea of the variables separation through the Hamiltonian principle has been generalized by Zhong [19], and by Xu et al. [20].

More recently, Sun et al. [21] employed a symplectic method yielding the Hamiltonian canonical equations in dual variables while studying the dynamic buckling of cylindrical shells under an axial impact. Both critical load and buckling mode are found by solving the symplectic eigenvalues and the associated solutions, respectively.

There is also another competing approach to omit drawbacks given by a direct/standard use of FEM incremental-iterative analysis, which is computationally expensive and sometimes difficult in the results interpretation. There exist a large record of papers and monographs devoted to semi-analytical and asymptotic/perturbation approaches devoted to study stability and buckling phenomena of structural members (for example see monographs [22–24]). There are also combined approaches, which for a sake of simplicity are not referenced here, focused on applications of perturbation analysis method with numerically-based approaches.

For instance, Schokker et al. [25] employed a perturbation technique for dynamic buckling analysis of composite cylindrical shells using the p-version of FEM, whereas Rahman and Jansen [26] applied the perturbation-based reduction procedure to study dynamic buckling of shell structures, which has been implemented to FEM code.

More recently, Fan et al. [27] analyzed the critical dynamic buckling load of cylindrical shells with arbitrary axisymmetric thickness variation with time-dependent uniform external pressure. Matching the Fourier series expansion, the regular perturbation method and the Sachenkov-Baktieva method, the analytical formulas have been derived governing the critical buckling load.

The geometrically non-linear sandwich shell theory introduced by Hohe and Librescu [28,29] has been utilized to study of load-frequency interaction phenomena in the dynamics buckling response of soft-core sandwich plates/shells by Hohe [30].

More recently, Paimushin [31] investigated a possibility for simplification the refined linearized equations of perturbed motion to identify the buckling mode shapes of isotropic spherical shells under external hydrodynamic pressure.

Paulo et al. [32] analyzed numerically the aluminium stiffened panels subject to axial compression with respect to the initial geometrical imperfections and material properties.

The so far described state-of-the-art of the investigation of buckling and postbuckling regimes of shells indicates a need of further research on this topic, in particular, when bifurcation and chaotic phenomena are taken into account. The originality and difference of our paper in comparison to the so far discussed works includes the following aspects. First, on the contrary to the referenced papers, we have included the physical non-linearity based on the Ilyushin theory of plastic deformations. The problem of dynamic buckling (new dynamic buckling criteria has been proposed) and other bifurcations has been coupled with the chaos theory (numerous novel computational results of non-linear dynamical behaviour of the spherical shells subject to harmonic load have been detected, illustrated, and discussed).

Second, contrarily to the mentioned papers aimed at sudying non-linear dynamics and buckling of shells using the perturbation methods, we have employed the FDM (Finite Difference Method) and the BGM (Bubnov-Galerkin method in higher approximation) putting emphasis on the analysis of convergence results as well as the results validity and reliability. In other words, our results are not limited to a few degrees of freedom, as it takes place while using asymptotic/perturbation/multiple scale methods, but we consider the studied shell as a continuous object with an infinite number of degrees of freedom.

Furthermore, we have employed novel numerical techniques and characteristics to study non-linear PDEs governing dynamics of rectangular spherical shells, which exhibits a difference between the present and the so far mentioned papers [33–37].

The paper is written in the following way. In Section 2 main assumption of the further investigated shell models are given together with associated stress-strain equations. The Hamiltonian variational principle yielding the governing PDEs and the boundary conditions is presented in Section 3. Method of solution via FDM (finite difference method) is illustrated in Section 4. The results reliability is discussed in Section 5, whereas the convergence analysis is carried out in Section 6. Scenarios of transition from regular to chaotic vibrations are detected, illustrated and discussed in Section 7 for different types of boundary conditions. The particular emphasis is devoted to shell buckling phenomena under time dependent load. Section 8 is devoted to concluding remarks regarding the carried out research.

2. Main assumptions

Let us consider a shallow shell of a rectangular form, having dimensions a, b, h along axes x_1, x_2, x_3, respectively. The origin of the coordinate system is located in the left top corner of the shell, in its mid-surface. Axes x_1, x_2 are parallel to the edges of the shell, while the axis x_3 is directed inward the curvature. In this specified system of coordinates the shell is defined as a three-dimensional area Ω, as follows: $\Omega = \{x_1, x_2, x_3 : (x_1, x_2, x_3) \in [0, \alpha] \times [0, \beta] \times [-h/2, h/2]\}$ (see Fig. 2.1).