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Stress based topology optimization of reinforcement structure under in-plane load

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ABSTRACT

As the static safety of mechanical structure is one of important criteria in engineering design process, it has been one of important topics to consider the static failure of a structure in topology optimization (TO). With the help of some recent relevant researches, some difficult issues in considering static failure are solved. However, this research found that the singularity issue which refers the difficulty of obtaining global optima with the KKT condition is not serious and mathematically relaxed for reinforcement TO design. And it is found that the existing qp-relaxation stress interpolation scheme to resolve the singularity issue in TO just shows the local optima issue in reinforcement TO design with different penalization factors in Solid Isotropic Material with Penalization (SIMP). In order to explain this feature, the TO problems for simple benchmark truss structures are revisited. Several two-dimensional examples only with in-plane load are solved to confirm the validity of the present study.

1. Introduction

This research considers the reinforcement design by topology optimization (TO) considering the stress failure constraint. After the development of TO, it has been widely applied for various engineering applications from microstructures to megastructures and from single physics system to multi-physics systems [1–10]. However, the stress based topology optimization problem (STOM) minimizing volume subject to local stress constraints has been considered as one of the most difficult problems due to the singularity issue, the many constraint issue and the highly nonlinear constraint issue. With the help of many important contributions, nowadays, it is possible to consider these stress constraints in TO and the stress constraints in multi-physics system. However, STOM is still regarded as one of important engineering problems and it should be extended to consider fatigue constraint [4,11–37]. One may think that this STOM also can be applied to the reinforcement design which finds out an optimal reinforcement design to constrain the maximum stress value. But applying STOM for the reinforcement design is not clear about the three issues: the singularity issue, the many constraint issue and the highly nonlinear constraint issue Particularly we found that the singularity issue needs an in-depth study and the role of the existing qp-relaxation method devised by many important relevant researches for the STOM also needs an in-depth investigation whether that method is valid or not for the STOM designing reinforcement structures [11,12,14,16,17,24,30,31,36–41].

The reinforcement structure increasing structural safety by adding some materials (reinforcements) to basic structures is common in civil and mechanical designs [42,43]. One merit of the usage of the reinforcement structure may be the non-destruction of basic structure and the increase of the safety. However, the reinforcement requires some extra materials and costs and the improper choice of the reinforcement structure can cause extra damages to basic structures. For examples, steel rib walls in ship building also can be regarded as reinforcements too (see Fig. 1).

To conduct the STOM minimizing volume subject to local stress constraints, it is very important to use the *qp*-relaxation method for the singularity issue with the *P*-norm approach for the local behavior of the stress constraints. The *qp*-relaxation method adopts the different penalization factors for the Young's modulus in the forward analysis and in the stress evaluation analysis; we do not have to limit to the penalization factors of the SIMP polynomial functions. Without this *qp*-relaxation method, the nominal stress values are decreased with smaller design values and what a gradient based optimizer face for TO with void and solid domains is that no-structure becomes a global optimum. In case of the reinforcement design, we found that this argument becomes unclear

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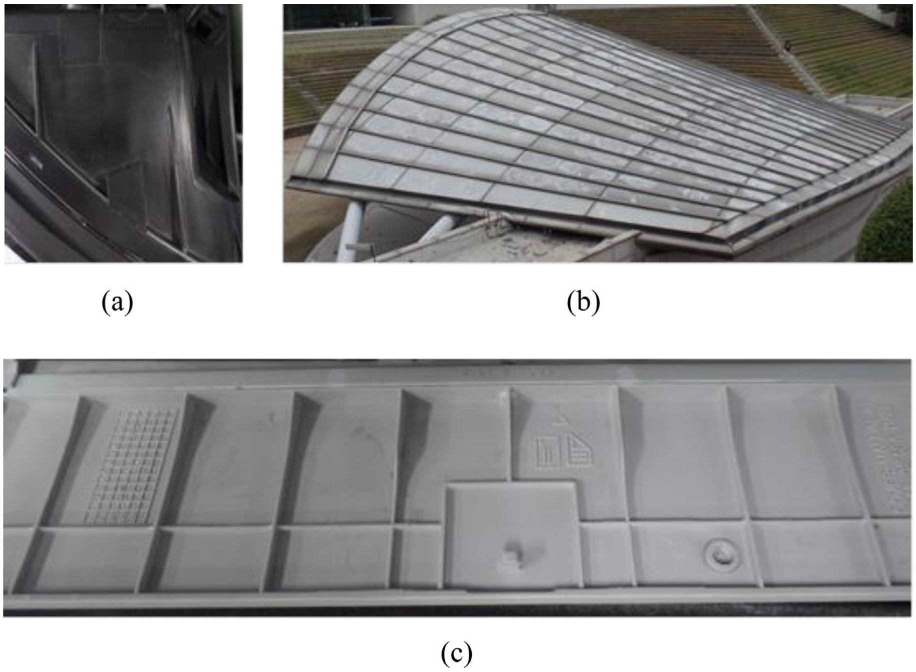

(a) (b) (c)

Fig. 1. Reinforcement structures in some architectures and machines: (a) rib structure in the automotive bonnet, (b) rib structure attached to roof and (c) plastic rib structures used in electronic device.

and obscure because of the existing of the basic (or base) structure in Fig. 1. To our best knowledge, the stress behaviors of reinforcement structures are not rigorously studied in TO. To address this unclear point, this research reinvestigates the singularity issue and the idea of the *qp*-relaxation method. In the present study, the in-plane load is only considered but the conclusions and findings can be applied for reinforcement structure with a combination of different types of loads.

The paper is organized as follows: In Section 2 and Section 3, the finite element formulation for reinforcement design is presented. The new singularity issue of reinforcement design with the existing *qp*-relaxation method is studied and some examples will be presented. Section 4 will provide several optimization examples to show the validity and effectiveness of the present stress interpolation issue. Finally, our findings are summarized in the conclusion.

2. Reinforcement design with finite element method and non-singularity issue

2.1. 2-dimensional finite element model with in-plane load

This section introduces a brief FE analysis for the reinforcement structure design considering the local stress constraints. To consider it in the FE framework, the following static finite element equations are employed [44]. Note that the thicknesses of reinforced structures can be increased and the central plane of rib may not be coplanar with the central plane of the original structure, *i.e.*, Fig. 1(c). Then the plane stress distribution in the rib along thickness direction may not be uniform even with in-plane load. The non-uniform stress distributions due to this side effect should be considered and this issue becomes critical in topology optimization when the stress distributions along thickness direction are not negligible for layerwise structure or functionally graded material (FGM) characterized by the variation in composition and structure gradually over volume.

$$\mathbf{KU} = \mathbf{F} \tag{1}$$

$$\mathbf{K} = \mathbf{K}_{\mathrm{R}} + \mathbf{K}_{\mathrm{B}} = \sum_{e=1}^{NE} (\mathbf{k}_{\mathrm{R},e} + \mathbf{k}_{\mathrm{B},e}) \tag{2}$$

$$\mathbf{k}_{\mathrm{R},e} = \iiint_V \mathbf{B}^{\mathrm{T}}\mathbf{C}_{\mathrm{R},e}\mathbf{B}dV, \mathbf{k}_{\mathrm{B},e} = \iiint_V \mathbf{B}^{\mathrm{T}}\mathbf{C}_{\mathrm{B},e}\mathbf{B}dV \tag{3}$$

$$\mathbf{C}_{\mathrm{R},e} = \gamma_e^n \mathbf{C}_0, \mathbf{C}_{\mathrm{B},e} = \mathbf{C}_0 \tag{4}$$

$$\text{Plane stress}: \quad \mathbf{C}_0 = \frac{E_0}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} \tag{5}$$

where the global stiffness, the global displacement, and the global force vector are denoted by **K**, **U** and **F**, respectively. The global stiffness matrix is further decomposed into the two terms, $\mathbf{K}_{\mathrm{R}}$ for the reinforcement structure and $\mathbf{K}_{\mathrm{B}}$ for the basic structure as shown in Fig. 2. The *e*-th elementary stiffness matrices for the reinforcement structure and the basic structure are $\mathbf{k}_{\mathrm{R},e}$ and $\mathbf{k}_{\mathrm{B},e}$, respectively. The design variable, γ_e, is assigned with the SIMP (Solid Isotropic Material with Penalization) penalization, *n* to interpolate the constitutive matrix, $\mathbf{C}_{\mathrm{R},e} = \gamma_e^n \mathbf{C}_0$, for $\mathbf{k}_{\mathrm{R},e}$.

2.2. Non-singularity problem of the reinforcement in simple truss design problem

From our best knowledge, there is no precedent research about the singularity issue in reinforcement design. Therefore, this subsection investigates the singularity issue in the truss reinforcement design. The singularity issue in the stress based TO refers the difficulties in numerically finding the global optimum with the KKT condition [12]. To resolve this difficulty, many researches have been conducted [4,11–25,27–37].

With basic structure and reinforcement structure, each structure can have different stress interpolation functions, i.e., here the different penalization factors of the SIMP based interpolation function. In other words, the stresses are evaluated by (6).