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Finite element analysis of finite strain micromorphic Drucker-Prager plasticity

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ABSTRACT

Earlier, Isbuga and Regueiro (2011) and Regueiro and Isbuga (2011) presented three dimensional finite element analysis of finite strain micromorphic isotropic elasticity based on the approach of Eringen and Suhubi (1964). We present the extension of this work to plasticity, following the formulation of Regueiro (2009, 2010) and Isbuga (2012). We assume the existence of an intermediate configuration and apply the separate multiplicative decomposition of the deformation gradient tensor and the micro-deformation tensor. In this paper, we investigate the effect of elastic length scale together with the boundary layer effect on micro-displacement tensor field for uniaxial strain and plane strain conditions, involving elastoplasticity with a Drucker-Prager yield function. We emphasize the importance of the additional degrees of freedom introduced by the micromorphic continuum formulation.

1. Introduction

Eringen and Suhubi [7] introduced the micromorphic continuum in which each material point is endowed with nine additional kinematic degrees of freedom (dof) that represents the most general case of higher order continua of "grade one" [5]. Isbuga and Regueiro [16] and Regueiro and Isbuga [28] were the first to extend this approach to three dimensional (3D) finite strain finite element analysis. They demonstrated the formulation and implementation for a finite strain micromorphic materially linear isotropic elasticity model. One advantage of a micromorphic continuum with regard to material constitutive modeling is that the additional degrees of freedom can be used in a multiscale approach to take into account the underlying micro-structure, as proposed by Regueiro [25,26], Regueiro and Yan [27]. In that approach, the additional degrees of freedom will incorporate the contribution of the lower length scale model, e.g., a discrete element method (DEM) for particulate materials. The micromorphic continuum accounts for micro-rotation, micro-shear, and micro-dilation/compaction of grain clusters in particulate materials, for instance, as opposed to just particle rotations in a micro-polar theory (see Gardiner and Tordesillas [9], Peters [24], Walsh and Tordesillas [37] as a small sampling of the broader literature on micropolar theories developed for granular media). Another main reason to employ the micromorphic continuum in multiscale modeling is that all stress tensors, together with body force vectors and body force couple tensors, can be expressed in terms of micro-scale tensors and parameters through integral averaging [5]. Besides the use of a micromorphic continuum in the context of multiscale modeling, which is the main interest of the paper, higher order continuum models because of their inherent nonlocality have been employed to overcome loss of ellipticity of the governing equations due to strain softening plasticity for modeling shear band localization, where the inherent length scale is found to regularize the governing equations. In addition, the application of higher order continuum models, including micromorphic and micropolar continua, to different types of material modeling such as concrete, metals, and soils exist in the literature [1,20,22,36,11,38]. The contribution of the paper is a finite strain treatment of Drucker Prager plasticity in the context of Eringen's general micromorphic theory.

Recent work on finite strain micromorphic continuum-based inelastic constitutive modeling follows different approaches, and can be summarized as follows.

Sansour et al. [30] presented an inelastic formulation for the micromorphic continuum at finite strain by following the previous work [29] on viscoplasticity. They mentioned the importance of the additional degrees of freedom, and defined a generalized deformation gradient, analogous to the micro-element deformation gradient $\boldsymbol{F}'$ in Regueiro [26], for which they applied the multiplicative decomposition. This is a departure from the approach we take in this paper, and also that in Forest and Sievert [8], that the

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multiplicative decomposition is applied separately to the deformation gradient $\boldsymbol{F}$ and micro-deformation tensor $\boldsymbol{\chi}$. By applying the multiplicative decomposition to the combined micro-element deformation gradient $\boldsymbol{F}'$, rather than separately to $\boldsymbol{F}$ and $\boldsymbol{\chi}$, we would loose the independence of defining constitutive equations for the various elastic and plastic parts of the deformation tensors. Thus, for now, we continue with our approach as outlined in Regueiro [25,26], motivated by Forest and Sievert [8], with the goal of concurrent multiscale modeling in mind Regueiro and Yan [27]. In addition, Sansour et al. [30] considered a higher grade ("grade two") micromorphic continuum with richer coordinates for the micro-space. They eventually simplified the generalized coordinate space to that of the macro-scale, three-dimensional, for computational implementation reasons. Vernerey et al. [36] proposed a method for modeling hierarchical materials within the context of a micromorphic continuum approach. They applied the method of virtual power (Germain [10]) to derive the weak form of the momentum equation, and incorporated plasticity. Forest and Sievert [8] constructed a general framework for elastoviscoplastic constitutive modeling of generalized continua, from which we base our separate multiplicative decomposition of $\boldsymbol{F}$ and $\boldsymbol{\chi}$. Dingreville et al. [2] investigated the wave propagation and dispersion character in 1D for elasto-plastic microstructured materials by following an approach proposed by Mindlin [21]. Researchers noted the importance of the length scale and microstructural material properties in overcoming the ill-posedness of the governing partial differential equations of wave propagation. Grammenoudis et al. [12] proposed a theoretical formulation of finite deformation plasticity with multiplicative decomposition for a micromorphic continuum coupled with damage. The formulation in Grammenoudis et al. [12] is most closely related to Regueiro [26], unbeknownst to the second author (Regueiro) when writing Regueiro [25,26], given almost the concurrent review and publication of these papers [12,25,26]. We focus, however, on a finite strain micromorphic Drucker-Prager plasticity model, and finite element implementation and results in this paper. Thus, this paper follows our previous papers on finite strain micromorphic elastoplasticity (Regueiro [25,26], Isbuga and Regueiro [16], Regueiro and Isbuga [28]) that involve a large deformation Total Lagrangian three-dimensional finite element implementation in the opensource C++ code Tahoe (`tahoe.sourceforge.net`). We believe we follow more directly Eringen's finite strain micromorphic "grade one" continuum formulation (as opposed to Germain [10], and other works derived thereof) extending to elastoplasticity using the multiplicative decomposition of Lee and Liu [19], Lee [18], similar to Grammenoudis et al. [12]. Therefore, in this work, we present the extension of Eringen and Suhubi [7]'s finite strain micromorphic isotropic elasticity to Drucker-Prager plasticity formulated in the intermediate configuration $\bar{\mathcal{B}}$ (Fig. 1) (see Regueiro [26], Isbuga [15] for more details) to be used in a future multi-scale approach described in Regueiro [25,26], Regueiro and Yan [27]. We assume two different yield criteria: (1) a Drucker-Prager (DP) yield function which involves no micromorphic terms; and (2) a Combined DP-like yield function (CDP) that involves the combination of the unsymmetric Cauchy stress tensor and relative stress tensor measures. We apply the multiplicative decomposition of the deformation gradient $\boldsymbol{F}$ and micro-deformation tensor $\boldsymbol{\chi}$ that assumes the existence of an intermediate configuration in which plastic multipliers are obtained by solving the linearized form of the DP and CDP yield functions within a Newton-Raphson nonlinear solution algorithm. The global consistent tangent is formed by linearizing the constitutive equations, including the dependence of the plastic multipliers on the displacement vector $\boldsymbol{u}$ and micro-displacement tensor $\boldsymbol{\Phi}$.

We assume Cartesian coordinates, and use a mix of index notation and symbolic/direct notation, such that $\boldsymbol{ab} = a_{ij}b_{jk}\boldsymbol{e}_i \otimes \boldsymbol{e}_k$,

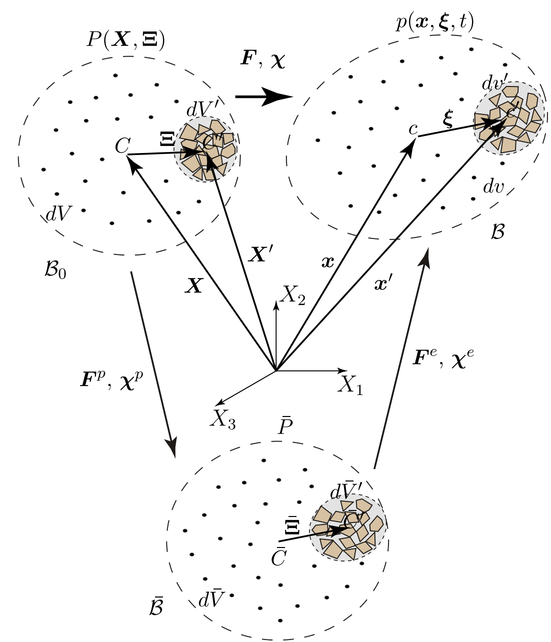

Fig. 1. Multiplicative decomposition of the deformation gradient $\boldsymbol{F}$ and micro-deformation tensor $\boldsymbol{\chi}$. Geometrical points ("macro-elements") with centroids $C, \bar{C}$, and c live in their respective configurations: reference $P \in \mathcal{B}_0$, intermediate $\bar{P} \in \bar{\mathcal{B}}$, and current $p \in \mathcal{B}$. Material points ("micro-elements") with centroids $C', \bar{C}'$, and c' also reside in these configurations, offset by relative position vectors $\boldsymbol{\Xi}, \bar{\boldsymbol{\Xi}}$, and ξ. Differential line elements and relative position vectors are mapped accordingly: $d\boldsymbol{x} = \boldsymbol{F}d\boldsymbol{X}, d\boldsymbol{x} = \boldsymbol{F}^e d\bar{\boldsymbol{X}}, d\bar{\boldsymbol{X}} = \boldsymbol{F}^p d\boldsymbol{X}, \xi = \boldsymbol{\chi}\boldsymbol{\Xi}, \xi = \boldsymbol{\chi}^e\bar{\boldsymbol{\Xi}}$, and $\bar{\boldsymbol{\Xi}} = \boldsymbol{\chi}^p\boldsymbol{\Xi}$.

where boldface denotes a tensor or vector, and e_i is the Cartesian base vector. Generally, variables in uppercase letters live in the reference configuration $\mathcal{B}_0$ (such as the reference differential volume dV), variables in uppercase with overbar letters live in the intermediate configuration $\bar{\mathcal{B}}$ (such as the intermediate differential volume $d\bar{V}$), and variables in lowercase live in the current configuration $\mathcal{B}$ (such as the current differential volume dv). The same applies to their indices, such that a differential line segment in the current configuration dx_i is related to a differential line segment in the reference configuration dX_I through the deformation gradient: $dx_i = F_{iI}dX_I$ (Einstein's summation convention is assumed; see Eringen [3], Holzapfel [13]). Subscripts $(\bullet)_{,i}$, $(\bullet)_{,\bar{I}}$ and $(\bullet)_{,I}$ imply partial differentiation with respect to the current, intermediate, and reference configurations, respectively, assuming Cartesian coordinates for the finite element implementation.

An outline of the remainder of the paper is as follows: Section 2 summarizes the yield functions, constitutive equations, and evolution equations in the intermediate configuration $\bar{\mathcal{B}}$; Section 3 formulates the weak form and finite element equations associated with the coupled momentum balance equations; Section 4 presents the numerical examples; and Section 5 provides conclusions.

2. Yield functions, constitutive equations, and evolution equations

In this section, we summarize the yield functions, constitutive equations, and evolution equations in the intermediate configuration $\bar{\mathcal{B}}$. The constitutive equations for linear isotropic elasticity were originally proposed by Eringen and Suhubi [7], Suhubi and Eringen [34] (see Table 1). Regueiro [25,26] presented the constitutive equations, the Clausius-Duhem inequality together with the derivation of the evolution equations in the intermediate configuration for J_2 flow plasticity Regueiro [26], and Drucker-Prager