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Coupled normal-shear stress models for SMA response

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ABSTRACT

In the present paper two constitutive models, able to reproduce the mechanical behavior of shape memory alloy (SMA), are proposed. The constitutive laws allow considering the simultaneous presence of normal and shear stresses, accounting for its coupling without developing a full three-dimensional (3D) model. In such a way, two different models suitable for deriving the response of SMA devices, both characterized by the presence of only normal and shear stresses, are presented. In particular, the proposed SMA models reproduce the pseudoelastic behavior, the shape memory effect, the reorientation process and account for the different elastic properties of austenite and martensite. The phase transformations are governed by an equivalent stress defined as function of the normal and shear stresses. A robust numerical algorithm, based on a backward Euler time integration within a predictor–corrector technique, is developed for each model. Numerical simulations of experimental evidences, available in literature, are performed to validate the proposed models and computational strategies. In particular, comparisons of the results obtained by the proposed models with experimental data, are performed. A comparison with a 3D model is also carried out. The ability of the proposed models in satisfactorily reproducing the SMA response with reduced computational cost is verified even for complex loading–unloading histories.

1. Introduction

Shape memory alloys (SMAs) are active materials able to undergo reversible large deformations when subjected to stress-temperature loading histories. Because of their unique properties and mechanical response, SMAs are suitable for a wide range of applications in mechanical, electrical, civil, medical and aerospace engineering.

The mathematical modeling of the very special thermomechanical response of SMAs represents an important issue for designing new applications and performing virtual testing of SMA devices.

Several one-dimensional (1D) and three-dimensional (3D) phenomenological models, able to reproduce the SMA constitutive behavior, have been proposed in the past decades. An update state of art of SMA modeling can be found in [1–3].

Usually SMA devices are geometrically characterized by one dimension significantly greater than the other two. For this reason, their mechanical behavior could be properly reproduced using 1D models. As a consequence, many efforts have been devoted to model and to predict the mechanical response of SMA in 1D framework. For example, Tanaka et al. [4] proposed a 1D model able to reproduce the SMA behavior introducing an internal variable representing the fraction of single-variant martensite. The Tanaka model does not consider the single-variant multivariant martensite phase transformation. Brinson [5] formulated a model based on the introduction of the martensite fraction internal state variable, which is assumed to be obtained as the sum of two parts: the fraction of the material that is purely temperature-induced martensite with multiple variants, and the fraction of the material that has been transformed by stress into a single martensitic variant. During the phase transformations, the evolution of the internal variables is governed by a cosine hardening law and the SMA elastic stiffness, evaluated by Voigt homogenization, is not constant. Bekker and Brinson [6] presented a model based on the thermomechanical paths in the stress-temperature space introducing cosine hardening function for describing the evolution of the internal variables during the phase transformation. Auricchio and Lubliner [7] presented a 1D model in the framework of general plasticity, considering pseudoelastic and shape memory effects. Auricchio and Sacco [8,9] presented a pseudoelastic 1D constitutive model, based on the introduction of one internal variable corresponding to the single-variant volume fraction. The evolution of the internal variable is governed by the stress and the temperature. Different homogenization techniques are considered to derive the elastic SMA modulus from the austenite and martensite elastic moduli. Auricchio and Sacco [10] proposed a SMA model able to describe

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pseudoelastic and shape memory effects, based on the introduction of two internal variables corresponding to the volume fractions of the single-variant martensite and of the multivariant martensite. The evolution of the internal variables is governed by the strain and temperature. A Reuss homogenization is considered to evaluate the elastic modulus of SMA, taking into account different elastic properties for the austenite and martensite. Govindjee and Kasper [11] developed a model based on the introduction of the martensite variants in tension and in compression considering constant elastic properties. They accounted for the plastic behavior of the material at high values of stress introducing a linear isotropic hardening plastic model and considering the evolution of the plastic variables independent on the phase transformations. Marfia et al. [12] proposed a 1D model for laminated shape memory alloy beam. Paiva et al. [13] formulated a model based on the introduction of four phases: tensile and compressive martensite, austenite, and multivariant martensite; a free energy of the SMA obtained as a combination of the free energy of the four phases is introduced. They also considered the presence of plastic deformations. Buravalla and Khandelwal [14] proposed a phenomenological 1D SMA model considering the elastic properties depending on the martensite fraction, assumed as an internal variable. Auricchio et al. [15] presented a phenomenological model that takes into account tension-compression asymmetries and elastic properties depending on the phase transformation evolution. Zbiciak [16] presented a formulation of the dynamic problem for Bernoulli-Euler beam made of pseudoelastic shape memory alloy. Marfia and Rizzoni [17] and Rizzoni and Marfia [18] proposed a thermodynamically consistent one-dimensional model based on three phases: austenite, compressive and tensile martensite, able to take into account the tension-compression asymmetries, the different elastic properties of the phases, the pseudoelastic and shape memory effects and the reorientation process.

One-dimensional SMA constitutive laws, accounting for the presence of only a normal stress, are successfully adopted in the development of several beam model and in its finite element implementation. They allow getting satisfactory results when beams are subjected mostly to flexural actions. Nevertheless, SMA devices are often not straight or are subjected even to torsional and shear loading histories; in these cases, significant shear deformations arise in SMA devices. For this reason, simulations of SMA devices subjected to complex loading histories are often performed adopting full 3D models.

Several 3D SMA models have been proposed in literature. Among the others, Raniecki et al. [19] proposed a model that takes into account the pseudoelastic behavior of polycrystalline SMAs. Liang and Rogers [20] discussed a constitutive model based upon the combination of both micromechanics and macromechanics to describe the phase transformation process. Graesser and Cozzarelli [21] proposed a SMA model taking into account the pseudoelasticity and the inelastic strain in a rate-type formulation similar to viscoplastic law. Other SMA models, available in the literature, have been proposed in [22–35]. It could be pointed out that it is not always an easy task to properly set the values of the material parameters governing these 3D models in order to predict the mechanical response when SMA is subjected to complex loadings inducing simultaneous transformation and reorientation processes. Moreover, stress analyses of SMA devices, developed adopting 3D finite elements, lead to expensive computational simulations. Most of the 3D models are deduced assuming the isotropic response of the material. On the other hand, SMA devices could present an anisotropic response; few SMA models accounting for the material anisotropy have been proposed, for instance, in [36,37], based on a micromechanical formulation.

The present study deals with the development of two new SMA constitutive models able to predict the mechanical behavior of the material subjected to normal and shear stresses. The SMA constitutive laws are characterized by simpler formulations with respect to the 3D ones and they are able to accurately account for all the significant features of the thermomechanical response of the SMA, introducing material parameters with a clear physical meaning. The models reproduce the pseudoelastic behavior, the shape memory effect and the reorientation process and consider different elastic properties of austenite and martensite and the anisotropic mechanical response of the material.

The normal and shear inelastic strain transformations are assumed as internal variables. The evolution of the internal variables during the phase transformations is governed by an equivalent stress, which takes into account the simultaneous presence of normal and shear stresses. Robust numerical algorithms, based on a backward Euler time integration are developed and the time step is solved adopting a predictor-corrector technique for both the proposed models.

Numerical applications are presented in order to assess the ability of the constitutive models in simulating the response of SMA devices. In particular, the numerical results, obtained adopting the proposed model, are compared with a 3D model and with experimental results, available in literature.

The paper is organized as follows. In Section 2, the proposed temperature-dependent SMA constitutive laws are presented; then, Section 3 describes in detail the computational procedures for the time integration of the evolutive constitutive equations; Section 4 deals with the numerical results; finally, concluding remarks are pointed out in Section 5.

2. Normal-shear stress SMA models

In this section, two SMA models, which account for the presence of normal and shear stresses, are proposed. The constitutive laws are derived for predicting the behavior of SMA devices geometrically characterized by one dimension significantly greater than the other two. For these kind of devices, the beam model can be successfully adopted to investigate their mechanical response. Beam theories are characterized by the presence of a normal and shear stresses in a typical cross-section, resulting in a stress state simpler with respect to the three-dimensional case but, in the meantime, more complex with respect to the uniaxial state. In fact, the presence of shear stresses in the beam model, due to shear and torsional loading, can significantly influence the SMA phase transformations, anticipating and accelerating the austenite-martensite change of phase. As consequence, classical 1D SMA model could become inaccurate in this case, so that often 3D models are adopted to get accurate results.

Aim of the proposed models is to account for the normal and shear stress arising in a beam formulation. The proposed models consider a parent phase associated to no macroscopic inelastic strain (austenite or multivariant martensite) and a product phase associated to macroscopic inelastic strain (single-variant martensite); consequently, they are not able to distinguish between austenite and multivariant martensite, since both phases do not produce any macroscopic inelastic strain. In the following, it is used "martensite" to refer to the single-variant martensite.

The models are able to reproduce:

- the pseudoelastic behavior as well as the shape memory effect,
- the different elastic proprieties of martensite and austenite phases,
- the coupling among normal and shear stresses in forward and reverse transformations,
- the development of normal and shear inelastic strains,
- the anisotropic behavior of the material,
- the reorientation of the single-variant martensite.