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Geometrically nonlinear isogeometric analysis of functionally graded microplates with the modified couple stress theory

Hoang X. Nguyen [a], Elena Atroshchenko [b], H. Nguyen-Xuan [c,d,*], Thuc P. Vo [a,e,*]

[a] Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne NE1 8ST, United Kingdom
[b] Department of Mechanical Engineering, University of Chile, Santiago 8370448, Chile
[c] Department of Physical Therapy, Graduate Institute of Rehabilitation Science, China Medical University, Taichung 40402, Taiwan
[d] Department of Architectural Engineering, Sejong University, 209 Neungdong-ro, Gwangjin-gu, Seoul 05006, Republic of Korea
[e] Institute of Research and Development, Duy Tan University, 03 Quang Trung, Da Nang, Vietnam

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ABSTRACT

In this study, a new and efficient computational approach based on isogeometric analysis (IGA) and refined plate theory (RPT) is proposed for the geometrically nonlinear analysis of functionally graded (FG) microplates. While the microplates' size-dependent effects are efficiently captured by a simple modified couple stress theory (MCST) with only one length scale parameter, the four-unknown RPT is employed to establish the displacement fields which are eventually used to derive the nonlinear von Kámán strains. The NURBS-based isogeometric analysis is used to construct high-continuity elements, which is essentially required in the modified couple stress and refined plate theories, before the iterative Newton-Raphson algorithm is employed to solve the nonlinear problems. The successful convergence and comparison studies as well as benchmark results of the nonlinear analysis of FG microplates ascertain the validity and reliability of the proposed approach. In addition, a number of studies have been carried out to investigate the effects of material length scale, material and geometrical parameters on the nonlinear bending behaviours of microplates.

1. Introduction

Classical elasticity has been well established and played a crucial role in the development of the material models and structural responses in various engineering fields ranging from mechanical to bio-engineering. It fundamentally follows the Hooke's law assuming the linear relation between the force and the change in displacement via the stiffness of the body on which the force is applied. However, the classical elasticity fails to capture the size-dependent effects which occur in the small-scale structures. These effects, indeed, have been pointed out by Lam et al. [1] after conducting the experimental bending test of epoxy polymeric microbeams witnessing the bending rigidity was 2.4 times higher as the beam thickness declined from 115 μm to 20 μm. Those small-scale structures have created new challenges in modelling, for instance, when one attempts to investigate the structural behaviours of elements in micro- and nano-electro-mechanical systems [2,3], carbon nanotube actuators [4], space and bio-engineering [5]. In order to model the materials and structures, the small length scales and its interaction with other particles should be carefully considered. These challenges encourage researchers to focus on a new research topic of modelling of small structures and predicting their behaviours. The theories for more general descriptions of materials' response have been initially developed dating back to the 1960s with the early works of Mindlin [6–8] and Mindlin and Tiersten [9] who developed higher-order theories of elasticity. There has also been a surge of interest in the generalised continuum since then including the development of non-local theory, strain gradient theory and couple stress theory.

The non-local elasticity was initially proposed by Erigen [10] and Erigen and Edelen [11] who assumed that the stress of a point in an elastic body not only depends on the strain at that point but also, theoretically, at all other points in the continuum. While this theory considers the interactions between atoms, it also includes the internal length scale in the constitutive equations as a material parameter [12]. However, as pointed out by Reddy [13], the Eringen's theory appears to be not applicable for the structural

* Corresponding authors at: Department of Physical Therapy, Graduate Institute of Rehabilitation Science, China Medical University, Taichung 40402, Taiwan (H. Nguyen-Xuan). Faculty of Engineering and Environment, Northumbria University, Newcastle upon Tyne NE1 8ST, United Kingdom (T.P. Vo).

E-mail addresses: xuan.h.nguyen@northumbria.ac.uk (H.X. Nguyen), eatroshch@gmail.com (E. Atroshchenko), a28271@mail.cmuh.org.tw (H. Nguyen-Xuan), thuc.vo@northumbria.ac.uk (T.P. Vo).

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mechanic problems in which the von Kármán and kinetic energy are involved. Concerning the strain gradient theory, this theory which was pioneered by Fleck et al. [14,15] assumes that the strain energy density depends on the first and second order displacement gradients. In addition, this theory contains the rotation gradient tensor, dilatation tensor, and deviatoric stretch gradient tensor. These assumptions require three material length scale parameters to be used in the strain gradient theory. Mindlin and Toupin are among those who first introduced the ideas of couple stress theory [9,16]. According to their studies, the strength of the continuum body is governed by both strain and curvature. Since it is experimentally difficult to determine all two material length scale parameters as proposed in the classical couple stress theory, models with less number of those parameters were in need to develop. Yang et al. [17] proposed the modified couple stress theory (MCST) which requires only one material length scale parameter in deriving the constitutive equation. In addition, this modification includes a symmetric couple stress tensor. Owing to these striking features, the MCST is continuously developed and has its own extensive literature. Chen and Li [18] developed quadrilateral spline element for couple stress elasticity. The MCST has been also applied to investigate the behaviours of beams with different types of theories including Bernoulli-Euler [19,20], Timoshenko [21] and higher-orders [22,23]. Employing the MCST, a large number of works predicting plates' behaviours has been done for both linear and nonlinear analyses. Tsiatas [24] presented the static analysis of isotropic microplates based on the MCST. While Yin et al. [25] considered the vibrational responses of microplates, an investigation on the behaviours of Mindlin microplates for both stretching effects and bending has been conducted by Ma et al. [26]. Thai and Vo [27] paid their attention to the bending and vibration responses of size-dependent microplates. Reddy and his colleagues worked on the nonlinear finite element analysis (FEA) of FG microplates with different geometries and plate theories [28,29]. It is worth commenting that functionally graded material (FGM) is a class of composite material which often consists of two different constituents varying their properties smoothly from one surface to another. Furthermore, the FG body can be deliberately tailored to inherit advantageous mechanical and thermal properties from the constituents of which it is made. A ceramic-metal FGM, for instance, benefits from the higher thermal resistance and better ductility from ceramic and metal phases, respectively. In addition, FGMs avoid stress concentration and delamination phenomena which are severe drawbacks of lightweight laminated composites. These striking features enable this material to be widely applied in various engineering fields such as aerospace, nuclear power plant, and bio-engineering in which the high-performance beam [30,31], plate [32,33] and shell [34] elements are involved.

When attention is turned to plate structures, there is a well established body of work on the development of mathematical models. The most basic plate theory is the classical plate theory (CPT), also known as Kirchhoff-Love plate theory. This theory basically assumes that the cross section perpendicular to the mid-plane before deformation remains normal to the mid-plane after deformation. As CPT neglects shear deformations, it is applicable only for thin plates in which the ratios of length to thickness are large. The first-order shear deformation theory (FSDT), also known as Reissner-Mindlin plate theory, was developed taking into account the shear deformations. This advantageous feature enables FSDT to yield reliable results for both thin and thick plates. However, the shear locking phenomenon which creates higher stiffness is often cited as a drawback of this theory when the problems are solved numerically by means of lower-order FEM. In addition, FSDT fails to predict the distribution of shear strains and stresses through the thickness for structures with traction free surfaces. In order to address this issue, one may need to include shear correction factor while using FSDT. However, this is not a straightforward approach as the shear correction factor does not stay the same for different problems. In order to bypass those shortcomings, Reddy pioneered the third-order shear deformation theory (TSDT) [35] before Soldatos proposed the higher-order shear deformation theory (HSDT) [36]. By making further assumptions to the TSDT, Senthilnathan [37] developed refined plate theory (RPT) which requires only four variables compared to Reddy's original five-unknown theory. In the last few years, the studies of the behaviours of microplates employing MCST and different plate theories have been enriched with a wealth of numerical solutions and analytical approaches. Reddy and his colleagues have successfully developed finite element models to analyse the behaviours of microplates with and without nonlinearity [28,38,39]. Similarly, Zhang et al. [40] presented the bending, free vibration, and buckling analyses with MCST by means of C^0 finite element method. Concerning the analytical approaches, Thai and his colleagues [27,41] investigated the bending and vibration responses of the FG microplates based on the TSDT and sinusoidal plate models with MCST. A size-dependent refined plate model for FG microplates based on MCST have been employed to solve for the closed-form solutions by He et al. [42]. It should be noted that although the RPT which requires only four unknowns owns positive properties compared to other models, it requires C^1-continuity elements which may cause difficulty when the conventional FEA is involved to solve the problem.

Recently, a newly developed numerical method which is able to deal with higher-order elements was initially coined Isogeometric Analysis by Hughes et al. [43]. This method also bridges the crucial gaps between the computer-aided design (CAD) field and the analysis field as it employs the same basis functions for representing geometries and conducting analysis. With well established works on CAD technology, the basis functions which commonly are B-splines or Non-Uniform Rational B-splines (NURBS) are able to exactly represents geometries. Furthermore, by its nature, those functions are highly smooth and can serve as the approximation basis of the unknowns. Also, when combined with appropriate plate theory, IGA is able to avoid locking phenomena as well as other techniques such as strain smoothing [44]. These positive properties make the IGA outweighs traditional FEA in many cases, especially for numerical problems where the high-continuity elements are involved such as C^1 plate analysis using HSDT or RPT. The basics and review of IGA as well as its computer implementation could be found in the established literature including the excellent works of Cottrell et al. [45], Vuong et al. [46], de Falco et al. [47], and Nguyen et al. [48]. IGA is also widely applied to solve for mechanical and thermal behaviours of complex structures such as plates [49–52] and shells [53–56]. Although IGA-based nonlinear analysis for plates has been touched following the works of the researchers in the community including [57–59], there is no reports on the nonlinear analysis of small-scale plates for size-dependent effects using this robust numerical method.

In this study, in order to fill the existing gap in the literature, the nonlinear analysis of FG microplates by means of the IGA will be proposed. The MCST with only one material length scale parameter is employed to account for the size-dependent effects of the small-scale FG plates. Meanwhile, the four-unknown RPT is used to describe the generalised displacement field of the microplates. The bending responses with nonlinearity are then numerically solved by the proposed NURBS-based IGA in which the iterative Newton-Raphson algorithm is involved. It is worth commenting that although there is still room for the performance of the NURBS functions regarding geometry representation as the domains of the plates considered in this study are not of high